MATH 201.1 (Term 111)

Quiz 1 (Sects. 10.1-4) Duration: 20mn

Name: ID number:

1.)(3pts)Find a Cartesian equation of the curve $x = e^{2t} + 5$, $y = e^{-t} + 2$, $t \in \mathbb{R}$.
2.)(3pts)Find the equation of the tangent line to the curve $x = t^2 + 1$, $y = t^2 - 1$, at $t = 0$.
3.)(4pts)Find all the points of intersection of the polar curves $r = \cos 2\theta - \frac{1}{4}$ and $r = -\sin^2\theta$, $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$.

1°) $\begin{cases} x = e^{2t} + 5 \\ y = e^{-t} + 2 \end{cases}, \quad t \in \mathbb{R}$

From the second equation we have $e^{t} = \frac{1}{y-2}$

We substitute into the first equation, and we get

$x = \frac{1}{(y-2)2} + 5$

$\boxed{(y-2)^2 = \frac{1}{x-5}, \quad x > 5}$

2) $\begin{cases} x = t^2 + 1 \\ y = t^2 - 1 \end{cases}$

$\frac{dx}{dt} = 2t$

$\frac{dy}{dt} = 2t$

At $t = 0$, $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} = 0$

$\lim_{t \to 0} \frac{dy}{dx} = \lim_{t \to 0} \frac{2t}{2t} = 1$

At $t=0$, the coordinates of the point are $x_0 = 1$ and $y_0 = -1$

The equation of the tangente at the point $(1,-1)$ is

$y - y_0 = (x - x_0)$

$y + 1 = x - 1$

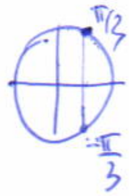
$\boxed{y = x - 2}$

3.) We solve $\cos 2\theta - \frac{1}{4} = -\sin^2\theta$,

$\cos^2\theta - \sin^2\theta - \frac{1}{4} = -\sin^2\theta$

$\cos^2\theta = \frac{1}{4}$

$\cos\theta = \frac{1}{2}$ or $\cos\theta = -\frac{1}{2}$



$\boxed{\theta = \frac{\pi}{3}, \frac{-\pi}{3}}$

MATH 201.5 (Term 111)

Quiz 1 (Sects. 10.1-4) Duration: 20mn

Name: ID number:

1.)(3pts)Find a Cartesian equation of the curve $x = \ln^2 t + 4$, $y = \frac{1}{\ln t} - 2$, $t > 1$.
2.)(3pts)Find the equation of the tangent line to the curve $x = e^t - t$, $y = e^t + t$, at $t = 0$.
3.)(4pts)Find all the points of intersection of the polar curves $r = \cos 2\theta$ and $r = \sin 3\theta$, $0 \le \theta \le \pi$.

1.) $\begin{cases} x = \ln^2 t + 4 \\ y = \frac{1}{\ln t} - 2 \end{cases}, t > 0$

From the second line, we deduce that

$$\ln t = \frac{1}{y+2}.$$

We substitute into the first equation. We find

$$x = \frac{1}{(y+2)^2} + 4$$

$$\boxed{(y+2)^2 = \frac{1}{x-4}, \quad x > 4}$$

2.) $\begin{cases} x = e^t - t \\ y = e^t + t \end{cases}$

$$\frac{dx}{dt} = e^t - 1$$

$$\frac{dy}{dt} = e^t + 1$$

at $t=0$, $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} = 2$

This is a vertical tangente.

The coordinates of the point at $t=0$ are $x_0 = 1$, $y_0 = 1$

So that the equation of the tangente line at $t=0$ is

$$\boxed{x = 1}$$

3.) We solve $\cos 2\theta = \sin 3\theta$, $\theta \in [0, \pi]$

$$\cos 2\theta = \cos\left(\tfrac{\pi}{2} - 3\theta\right)$$

$$\Rightarrow 2\theta = \frac{\pi}{2} - 3\theta + 2k\pi \quad \text{or} \quad 2\theta = -\frac{\pi}{2} + 3\theta + 2k\pi$$

$$5\theta = \frac{\pi}{2} + 2k\pi \quad \text{or} \quad -\theta = -\frac{\pi}{2} + 2k\pi$$

$$\theta = \frac{\pi}{10} + \frac{2k\pi}{5} \quad \text{or} \quad \theta = \frac{\pi}{2} + 2k\pi$$

$$\boxed{\theta = \frac{\pi}{10}, \frac{\pi}{2}, \frac{9}{10}\pi}$$