King Fahd University of Petroleum and Minerals
Quiz 1 Math 102-111 Duration 25 minutes

Question 1 Using four rectangles and right endpoints to approximate the area under the graph of $f(x) = \ln|x-1|$ from $x = -8$ to $x = 0$.
Solution

$$\Delta x = \frac{0-(-8)}{4} = 2,$$

and hence, we can estimate the area A as:

$$A \approx \Delta x\,(f(-6)+f(-4)+f(-2)+f(0))$$
$$= 2\,(\ln|-7| + \ln|-5| + \ln|-3| + \ln|-1|)$$
$$= 2(\ln 7 + \ln 5 + \ln 3) = 2\ln(105)\,.$$

Question 2 Write the given limit as a definite integral:

$$\lim_{n\to\infty}\sum_{i=1}^{n} \frac{\pi^3 i}{(2n)^2}\cos(\frac{\pi i}{2n}+1)\,.$$

Solution Let $x_i = \frac{\pi i}{2n}$ and so, $a = 0$ and $\Delta x = \frac{\pi}{2n}$. Then, $\frac{\pi}{2} = b - a \Longrightarrow b = \frac{\pi}{2}$. Now,

$$\frac{\pi^3 i}{(2n)^2}\cos(\frac{\pi i}{2n}+1) = \Delta x\,(\pi x_i \cos(x_i+1))$$

Therefore,

$$\lim_{n\to\infty}\sum_{i=1}^{n} \frac{\pi^3 i}{(2n)^2}\cos(\frac{\pi i}{2n}+1) = \int_0^{\frac{\pi}{2}} \pi x\cos(x+1)\,dx\,.$$

Question 3 Evaluate the integral

$$\int_0^{\sqrt{2}} \sqrt{4-2x^2}\,dx\,.$$

(You may interpreting it as an area.)
Solution

$$\int_0^{\sqrt{2}} \sqrt{4-2x^2}\,dx = \sqrt{2}\int_0^{\sqrt{2}} \sqrt{2-x^2}\,dx = \sqrt{2}\frac{\pi 2}{4} = \frac{\pi}{\sqrt{2}}$$

Question 4 Evaluate $f(0)$ if

$$3xf(0) + \int_x^{x^2} |t-1|f(t)\,dt = 5x\,.$$

Solution Differentiate both sides and using the FTCI, we get

$$3f(0) + |x^2-1|f(x^2)\,(2x) - |x-1|f(x) = 5\,.$$

Substitute $x = 0$, we obtain

$$3f(0) - f(0) = 5 \Longrightarrow f(0) = 5/2\,.$$

Question 5 If the velocity of a particle moving along a straight line is given by $v(t) = \frac{1}{2} - \cos t$, then find the distance traveled during the interval time $[0, \frac{\pi}{2}]$.
Solution Let d be the distance traveled during the time interval $[0, \frac{\pi}{2}]$. So,

$$
\begin{aligned}
d = \int_0^{\frac{\pi}{2}} |v(t)|\,dt &= \int_0^{\frac{\pi}{3}} (-v(t))\,dt + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} v(t)\,dt \\
&= \int_0^{\frac{\pi}{3}} (\cos t - 1/2)\,dt + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (1/2 - \cos t)\,dt \\
&= (\sin t - t/2)\Big|_0^{\frac{\pi}{3}} + (t/2 - \sin t)\Big|_{\frac{\pi}{3}}^{\frac{\pi}{2}} \\
&= \left(\sin(\frac{\pi}{3}) - \frac{\pi}{6}\right) + \left[\frac{\pi}{4} - \sin(\frac{\pi}{2}) - (\frac{\pi}{6} - \sin(\frac{\pi}{3}))\right] = \sqrt{3} - 1 - \frac{\pi}{12}
\end{aligned}
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