King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

CODE 001 | Math 102 | CODE 001

Math 102
Final Exam
Term 111
Monday, January 9, 2012
Net Time Allowed: 180 minutes

Name: ______________________________

ID: __________________ Sec: __________________.

Check that this exam has 28 questions.

Important Instructions:

1. All types of calculators, pagers or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. $\int_0^3 |4x - 8| \, dx =$

(a) 11

(b) 12

(c) 10

(d) 13

(e) 14

2. The series $\sum_{n=1}^{\infty} \left(\frac{6}{4n-1} - \frac{6}{4n+3} \right)$ is

(a) convergent and its sum is 2

(b) convergent and its sum is 1

(c) convergent and its sum is 0

(d) divergent

(e) convergent and its sum is 6

3. $\int_e^3 \frac{1}{x \ln \sqrt{x}} dx =$

(a) $\ln(\ln 3) - 2$

(b) $2\ln(\ln 3)$

(c) $\ln(\sqrt{\ln 3})$

(d) 1

(e) $-1 + 2\ln 3$

4. The first three terms of the Taylor series of $f(x) = \sqrt{1+3x}$ about $a = 1$ are given by

(a) $2 + (x-1) + \frac{1}{2}(x-1)^2$

(b) $2 + \frac{3}{4}(x+1) - \frac{9}{64}(x+1)^2$

(c) $2 + \frac{3}{4}(x-1) - \frac{9}{64}(x-1)^2$

(d) $2 + 3(x-1) - 9(x-1)^2$

(e) $1 - 2(x+1) + 3(x+1)^2$

5. If $F(x) = \int_x^0 \sqrt{1+t^3}\,dt$, then $F'(x) =$

(a) $1 - \sqrt{1+x^3}$

(b) $-\sqrt{1+x^3}$

(c) $\sqrt{1-x^3}$

(d) $\sqrt{1+x^3}$

(e) $1 + \sqrt{1+x^3}$

6. $\int_0^{\pi/4} \sec^4\theta \tan^4\theta d\theta =$

(a) $1/6$

(b) $1/12$

(c) $12/35$

(d) $2/57$

(e) $2/7$

7. The area of the region bounded by the curves $y = e^x, y = -x + 1, x = 1$ is equal to

(a) $e - \frac{3}{2}$

(b) e^2

(c) $3e - 2$

(d) $2e - 1$

(e) $e + 2$

8. $\int \frac{(1+x)^2 - 2x}{\sqrt{x}} dx =$

(a) $x^{-1/2} - \sqrt{x} + C$

(b) $2x^{-1/2} + \frac{5}{2}x^{2/5} + C$

(c) $2\sqrt{x} - 5x^{2/5} + C$

(d) $\sqrt{x} + 5x^{5/2} + C$

(e) $2\sqrt{x} + \frac{2}{5}x^{5/2} + C$

9. The sequence $\left\{\sqrt[n]{2^{3n-1}}\right\}_{n=2}^{\infty}$ is

(a) convergent and its limits is 4

(b) divergent

(c) convergent and its limits is $1/2$

(d) convergent and its limits is 8

(e) convergent and its limits is 2

10. The volume of the solid generated by rotating the region enclosed by the curves $y = x^2 - x$ and $y = 0$ about the line $x = 1$ is equal to

(a) $\frac{\pi}{2}$

(b) $\frac{\pi}{6}$

(c) π

(d) 4π

(e) $\frac{2\pi}{3}$

11. Which one of the following statements is **TRUE**: (C: Convergent;D:Divergent; AC: Absolutely Convergent; CC:Conditionally Convergent)

(a) $If \sum_{n=1}^{\infty} a_n$ is C, then $\sum_{n=1}^{\infty} a_n$ is AC.

(b) $If \sum_{n=1}^{\infty} a_n$ is CC, then $\sum_{n=1}^{\infty} |a_n|$ is D.

(c) $If \sum_{n=1}^{\infty} a_n$ is CC, then $\sum_{n=1}^{\infty} a_n$ is D.

(d) $If \sum_{n=1}^{\infty} a_n$ is CC, then $\sum_{n=1}^{\infty} a_n$ is AC.

(e) $If \sum_{n=1}^{\infty} a_n$ is AC, then $\sum_{n=1}^{\infty} |a_n|$ is D

12. The volume of the cone generated by revolving the triangular region with vertices $(0,0), (2,0), (2,3)$ about the x-axis is equal to

(a) 6π

(b) π

(c) $\frac{\pi}{3}$

(d) $\frac{4\pi}{3}$

(e) 4π

13. The series $\sum_{n=2}^{\infty} 3^{n+1} \cdot 2^{1-2n}$ is

(a) divergent

(b) convergent and its sum is $27/2$

(c) convergent and its sum is $9/4$

(d) convergent and its sum is $3/2$

(e) convergent and its sum is $27/8$

14. $\int_0^4 \sqrt{4x - x^2} dx =$

(a) π

(b) 2π

(c) $\pi/2$

(d) 3π

(e) $3\pi/4$

15. The sum of the convergent series $\sum_{n=1}^{\infty} \frac{\pi^{n-1}}{n!}$ is equal to

(a) $\frac{e^{\pi} - \pi}{\pi}$

(b) $\frac{e^{\pi} - 1}{\pi}$

(c) $\frac{e^{\pi}}{\pi} - 1$

(d) $1 - \frac{e^{\pi}}{\pi}$

(e) $\frac{e^{\pi}}{\pi}$

16. $\int \frac{x+4}{x^2 - x - 2} dx =$

(a) $\ln \left| \frac{(x-2)^2}{x+1} \right| + C$

(b) $\ln \left| \frac{x-2}{x+1} \right| + C$

(c) $\ln |x-2| + \ln |x+1| + C$

(d) $\ln |x^2 - x - 2| + C$

(e) $\ln |x-2| + 4 \ln |x+1| + C$

17. $\int \frac{1}{\sqrt{x} - \sqrt[4]{x}} dx =$

(a) $\sqrt{x} - 2\sqrt[4]{x} + 4\ln\left|\sqrt[4]{x} + 1\right| + C$

(b) $4\sqrt{x} + \tan^{-1}(\sqrt[4]{x}) + C$

(c) $2\sqrt{x} - 4\sqrt[4]{x^3} + C$

(d) $2\sqrt{x} + 4\sqrt[4]{x} + 4\ln\left|\sqrt[4]{x} - 1\right| + C$

(e) $4\sqrt[4]{x} + \tan^{-1}(\sqrt{x}) + 2\ln\left|\sqrt{x} + 1\right| + C$

18. The series $\sum_{n=1}^{\infty} (-1)^n \frac{(n!)^2}{(2n+1)!}$ is

(a) divergent by the comparison test

(b) divergent by the test for divergence

(c) divergent by the ratio test

(d) convergent by the ratio test

(e) a series with which the ratio test is inconclusive

19. The interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(2x+1)^n}{n}$ is

(a) $[-1, 1)$

(b) $(-1, 1]$

(c) $(-1, 0]$

(d) $[-1, 0]$

(e) $[-1, 0)$

20. In trying to apply **the integral test** to the series $\sum_{n=1}^{\infty} \frac{4n}{(n^2+1)^2}$, we conclude that

(a) the series is divergent.

(b) the series may converge or diverge.

(c) the series is convergent.

(d) the integral test is not applicable.

(e) the series is convergent and its sum is 1.

21. The series $\sum_{n=1}^{\infty} \sin\left(\frac{(-1)^n}{n}\right)$ is

(a) divergent by the Comparison Test.

(b) divergent by the Alternating Series Test.

(c) divergent by the Test for Divergence.

(d) convergent by the Limit Comparison Test.

(e) convergent by the Alternating Series Test.

22. The length of the curve $y = x^4 + \frac{1}{32x^2}$, $1 \le x \le \sqrt{2}$, is equal to

(a) $\frac{193}{64}$

(b) $\frac{73}{51}$

(c) $\frac{9}{64}$

(d) $\frac{5}{16}$

(e) $\frac{5}{32}$

23. If the curve $x = 2\sqrt{4-y}, 0 \le y \le 1$ is rotated about the $y-$axis, then the **surface area** of the generated solid is equal to

(a) $\frac{8\pi}{3}(5\sqrt{5}-8)$

(b) $\frac{\pi}{6}(8-5\sqrt{5})$

(c) $\frac{4\pi}{3}(5\sqrt{5}-2)$

(d) $\frac{8\pi}{3}(5\sqrt{5}-4)$

(e) $\frac{40\sqrt{5}}{3}\pi$

24. $\int_0^1 \sin(x^2)dx =$

(a) $\sum_{n=0}^{\infty} \frac{1}{(4n+3)!}$

(b) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)! \cdot (n+1)}$

(c) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)! \cdot (4n+3)}$

(d) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)! \cdot (3n+2)}$

(e) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}$

25. A power series representation for $f(x) = \dfrac{x}{16 - x^4}$ is given by (for $|x| < 2$)

(a) $\sum\limits_{n=0}^{\infty} \dfrac{x^{4n}}{16^n}$

(b) $\sum\limits_{n=0}^{\infty} \dfrac{x^{4n+1}}{2^{4n+4}}$

(c) $\sum\limits_{n=0}^{\infty} (-1)^n \dfrac{x^{4n+2}}{16^{n+1}}$

(d) $\sum\limits_{n=0}^{\infty} \dfrac{x^{4n+2}}{2^{4n+2}}$

(e) $\sum\limits_{n=0}^{\infty} (-1)^n \dfrac{x^{4n+1}}{16^n}$

26. $\displaystyle\int_2^3 \frac{1}{(1-x)^2} \ln\left(\frac{x+1}{x-1}\right) dx =$ (Hint: Make the substitution $u = \dfrac{x+1}{x-1}$.)

(a) 1

(b) $2 - 2\ln 2 + \ln 3$

(c) $\dfrac{1}{2}(3\ln 3 - 2\ln 2 - 1)$

(d) $\ln 2 - \dfrac{3}{2}\ln 3$

(e) $1 + \ln 2 + \ln 3$

27. $\int_0^{\pi^2} \cos(\sqrt{t})dt =$

(a) $\pi - 3$

(b) -4

(c) $2\pi + 2$

(d) -4π

(e) 2π

28. **Fill in the blank**: In applying the **Comparison Test** to the series $\sum_{n=1}^{\infty} \frac{\sin^2 n}{n\sqrt{n}}$, we use the series........ and conclude that the series $\sum_{n=1}^{\infty} \frac{\sin^2 n}{n\sqrt{n}}$ is........

(a) $\sum_{n=1}^{\infty} \frac{1}{n}$; divergent .

(b) $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$; convergent.

(c) $\sum_{n=1}^{\infty} \frac{1}{n^2}$; convergent.

(d) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$; divergent .

(e) $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$; divergent.