1. The function $f(x) = \begin{cases} ax^2 + bx & \text{if } x \leq 1 \\ x + a^2 & \text{if } x > 1 \end{cases}$

   is twice differentiable everywhere. Then $a^2 + b^2 =$

   a) 1
   b) 0
   c) $\frac{5}{4}$
   d) 2
   e) 5

2. If $f(x) = (2x-1)^{\frac{2}{3}}$, then the equation of the vertical tangent to the graph of $f$ is

   a) $x = \frac{1}{2}$
   b) $x = -\frac{1}{2}$
   c) $x = \frac{2}{3}$
   d) $x = -\frac{2}{3}$
   e) $x = \frac{4}{3}$

3. The equations of the horizontal tangents to the curve $y = x^3 - 3x - 2$ are

a) $y = 0$ and $y = -4$
b) $y = 1$ and $y = -1$
c) $x = 1$ and $x = -1$
d) $y = -4$ and $y = 1$
e) $y = 0$ and $y = -1$

4. At how many real values of $x$ does the curve $y = x^6 - 3\,x^2 + x + 5$ have a tangent line parallel to the line $y = x$?

a) 3
b) 1
c) 2
d) 4
e) 5

5. If the tangent line to the graph of $f(x) = \dfrac{2x}{2x+1}$ at the point $(\alpha, \beta)$ is $y = 2\,x + 1$, then $\beta^2 =$

a) 1
b) 2
c) 3
d) 4
e) 5

6. If $f(x) = xe^x$ and $n$ is a positive integer, then $f^{(n)}(1) =$

a) $(n+1)\;e$
b) $n\;e$
c) $(n-1)\;e$
d) $(n+2)\;e$
e) $n\;e+1$

7. If $y = \dfrac{1+\sin x}{1+\cos x}$, then $\dfrac{dy}{dx} =$

a) $\dfrac{1+\sin x+\cos x}{(1+\cos x)^2}$

b) $\dfrac{\sin x+\cos x}{1+\cos x}$

c) $\dfrac{\sin x+\cos x}{(1+\cos x)^2}$

d) $\dfrac{1+\sin x}{(1+\cos x)^2}$

e) $\dfrac{2}{1+\cos x}$

8. $\lim\limits_{\theta\to 1} \dfrac{\sin(\theta-1)}{\theta^2+\theta-2} =$

a) $\dfrac{1}{3}$

b) 0

c) $\dfrac{1}{2}$

d) 2

e) 1

9. If $y = \sin(x^2)$ and $x = \cos t$, then $\dfrac{dy}{dt} =$

a) $-\sin 2t \;\cos(\cos^2 t)$

b) $\sin 2t \;\cos(\cos^2 t)$

c) $-\sin t \;\cos(\cos^2 t)$

d) $-\sin 2t \;\cos^3 t$

e) $\sin 2t \;\cos^3 t$

10. Let $f$ and $g$ be differentiable functions and $h(x) = f(x^2 g(x))$. If $g(2) = -2$ and $g'(2) = 2$, then $h'(2) =$

a) $0$

b) $-2$

c) $2$

d) $3$

e) $-3$

11. The equation of the tangent line to the curve given implicitly by

$$\sqrt{x+y} = y^2$$

at the point $(0, 1)$ is

a) $3y - x = 3$
b) $2y + x = 1$
c) $3y + x = 3$
d) $2y - x = 2$
e) $2y + x = 3$

12. The equation of the normal line to the curve $y = \tan^{-1}\left(\sqrt{x-1}\right)$ at $x = 2$ is

a) $y = -4x + 8 + \dfrac{\pi}{4}$
b) $y = \dfrac{1}{4}x - \dfrac{1}{2} + \dfrac{\pi}{4}$
c) $y = 4x - 8 + \dfrac{\pi}{4}$
d) $y = -\dfrac{1}{4}x + \dfrac{1}{2} + \dfrac{\pi}{4}$
e) $y = -4x + 8 - \dfrac{\pi}{4}$

13. If $f(x) = (x^2 + 2\ x)^{50}$, then $f^{(100)}(1) =$

a) $100\,!$
b) $100$
c) $0$
d) $3(99\,!)$
e) $2(50\,!)$

14. The slope of the tangent line to the graph of $y = (2x+1)^{\sin 3x}$ at $x = \dfrac{\pi}{6}$ is

a) $2$
b) $4\left(\dfrac{\pi}{3} + 1\right)$
c) $6$
d) $2\left(\dfrac{\pi}{3} + 1\right)$
e) $\dfrac{4}{\frac{\pi}{3} + 1}$

15. If $y = \dfrac{(x+2)^2(2x-1)^3}{\sqrt{x+1}}$, then $y'(0) =$

a) 22
b) $-\dfrac{11}{2}$
c) 44
d) 24
e) $-11$

16. The position function of a particle moving along a line is

$$s(t) = \sin t + \cos t$$

where $t$ is measured in seconds and $s$ in meters. The total distance traveled by the particle in the interval $[0, \pi]$ is

a) $2\sqrt{2}$ meters
b) 2 meters
c) 4 meters
d) $2\sqrt{2} + 2$ meters
e) $2\sqrt{2} - 2$ meters

17. The position function of a particle moving along a line is

$$s(t) = t^3 - 6t^2 + 9t \quad (0 \le t \le 5).$$

The time interval(s) where the particle is moving forward is (are)

a) $(0,1)$ and $(3,5)$
b) $(0,3)$
c) $(0,3)$ and $(4,5)$
d) $(1,3)$
e) $(0,2)$ and $(3,5)$

18. The two equal sides of an isosceles triangle have length $4m$. If the angle between them is increasing at a rate of 0.06 $rad/s$, then the rate at which the area of the triangle is changing when the angle between the sides of the triangle is $\frac{\pi}{3}$ equals

a) $0.24 \ m^2/s$
b) $-0.24 \ m^2/s$
c) $2.4 \ m^2/s$
d) $-2.4 \ m^2/s$
e) $0.024 \ m^2/s$

19. If a snow ball melts so that its surface area decreases at a rate of 1 $cm^2/\text{min}$, then the rate at which the diameter changes, when the diameter is 10 $cm$ equals

Hint: Surface area of a sphere $= 4\pi r^2$

a) $\dfrac{-1}{20\pi}$ $cm/min$

b) $\dfrac{1}{20\pi}$ $cm/min$

c) $\dfrac{-1}{40\pi}$ $cm/min$

d) $\dfrac{1}{40\pi}$ $cm/min$

e) $\dfrac{-1}{10\pi}$ $cm/min$

20. The equation of the tangent line to the graph of $y = \ln x$ and passes through the origin is

a) $e\ y = x$

b) $y = e\ x$

c) $y = \dfrac{1}{e}(x - 1)$

d) $y = \dfrac{1}{e}(x + 1)$

e) $y = 2\ e\ x$