Math 590/690 (101)

Applied Fractional Calculus

Final Exam

Due on Wednesday, January 26, in room 59-2032.

(1) Let $\alpha > 0$, $n = [\alpha] + 1$, and $H(t)$ be the Heaviside function. Show that if $I_a^{n-\alpha} f \in C^n[a, b], a > 0$, then

$$D_0^\alpha \Big[H(t-a) f(t) \Big] \stackrel{a.e}{=} H(t-a) D_a^\alpha f(t), \qquad \alpha > 0.$$

What happens at $t = a$?

(2) Show that

$$D_0^\alpha \Big[\cosh\Big(\sqrt{\lambda} t\Big) \Big] = t^{-\alpha} E_{2,1-\alpha}(\lambda t^2), \qquad \alpha > 0.$$

(3) Find the fundamental set of the equation:

$$\Big[D^{4/3} - D \Big] \, y(t) = 0.$$

(4) Find an integral equation (equation with no derivatives) satisfied by the solution $y \in C[0, b]$ of the equation

$$\Big[D^{\sqrt{7}} + 2D^{3/4} \Big] \, y(t) = 0.$$
