MATH 572

Assignment 1   Due Date: November 02, 2010

Consider the following two-point BVP:

$$-u'' + 4u = (\pi^2 + 4)\sin(\pi x) \quad \text{for } 0 < x < 1 \text{ with } u(0) = u(1) = 0.$$

a) Verify that $u = \sin(\pi x)$ is a solution of the given BVP.

b) Use a uniform partition with **M** subintervals each of length **h** to define the piecewise linear FEM of the given BVP.

c) Write the proposed scheme (in part (b)) in a matrix form (find the elements of the obtained matrices).

d) Choose $\mathbf{M} = 20$. Compute the finite element solution at the nodes of the partition. Use the obtained datum to plot the approximate solution. Plot the exact solution on the same figure and compare between them.

e) Say that $u_\mathbf{h}$ is your piecewise linear finite element solution. Use the L_2-norm to compute the error between u and $u_\mathbf{h}$ for $\mathbf{M} = 20, 40, 80$ and 160. Find the order of convergence.