King Fahd University of Petroleum & Minerals
Department of Math. & Stat.

Math 568 - Midterm Exam (101)   Time: 2 hours 30

Dec. 14, 2010

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Name: ID #
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Problem 1	/10
Problem 2	/10
Problem 3	/5
Problem 4	/10
Problem 5	/8
Problem 6	/7
Total	/50

Problem # 1. (10 marks) Use the characteristic method to solve the problem

$$\begin{aligned} &yu_x + xu_y + u_z = u \\ &u(x, y, 0) = x^2 - y^2 \end{aligned}$$

Problem # 2. (10 marks) Use the characteristic method to solve the problem

$$\begin{aligned} &u_x^2 + yu_y - u = 0 \\ &u(x,1) = \tfrac{x^2}{4} + 1 \end{aligned}$$

Problem # 3. ***a*.** (5 marks) Use the Lagrange method to solve the equation

$$xu_x + yu_y = (x + y)u$$

b. Find the solution u which satisfies $u(x, -x) = 1$

Problem # 4. (10 marks) Given the partial differential equation

$$u_{xx} + 2xyu_{xy} + x^2y^2u_{yy} = -(x^2+1)yu_y$$

a. Show, by a convenient change of variable, that the equation can be reduced to

$$v_{vv} = 0$$

b. Solve the equation associated with the Cauchy data $u(0, y) = u_x(0, y) = y^2$

Problem # 5. (8 marks) Given the Cauchy problem

$$\begin{aligned} &u_{tt}(x,t) - 4u_{xx}(x,t) - \tfrac{8}{x}u_x(x,t) = e^t, \qquad x \neq 0, \quad t > 0 \\ &u(x,0) = x, \quad u_t(x,0) = 1 \end{aligned} \tag{P}$$

a. Let $v = xu$. Find the problem (P') satisfied by v.

b. Solve Problem (P')

c. Solve Problem (P)

Problem # 6. (7 marks) Given the Cauchy problem

$$\begin{array}{ll} u_{tt}(x,t) - c^2 u_{xx}(x,t) = 0, & -\infty < x < +\infty, \quad t > 0 \\ u(x,0) = 0, \quad u_t(x,0) = g(x), & -\infty < x < +\infty \end{array}$$

such that

$$g(x) = \begin{cases} \sin^2(x^2 - 1), & -1 < x < 1 \\ 0, & |x| \geq 1 \end{cases}$$

a. Show that the solution u is classical

b. Use the finite-speed propagation property to find $\lim_{x \to \mp\infty} u(x,t)$