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MATH. 533-101

Prob. 1
Find the images of the following
(i) The family of circles $C(a;|a|)$
(ii) The family of parallel lines $y = x + b$
(iii) The family of straight lines through z_0 $(z_0 \neq 0)$
(iv) The parabola $y = x^2$

Prob. 2
Evaluate $\int_0^{2\pi} f(re^{i\theta})d\theta$ in case f is analytic in $C(0;R)$ and $0 < r > R$

Prob. 3
Evaluate $\int_0^{2\pi} \log\left|re^{i\theta} - a\right| d\theta$, $r < |a|$

Prob. 4
Suppose that C is a closed, regular curve omitting the points $0, 1, -1$. Find all possible values of $\int_C \frac{dz}{z(z^2-1)}$

Prob. 5
Evaluate (a) $\int_{C(0;a)} \frac{zdz}{z^4-1}$, $a > 1$, (b) $\int_{C(0;2a)} \frac{e^z dz}{z^2+a^2}$, $a > 0$.

Prob. 6
Using Cauchy's formula for the derivative evaluate $\int_{C(0;r)} \frac{dz}{(z-b)(z-b)^m}$, $|a| < r < |b|$

Prob. 7
Express $|\sinh z|^2$, $|\cosh z|^2$ as functions of x and y

Prob. 8
Evaluate $Arctan(1+2i)$ and $Arctane^{i\theta}$, $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

Prob. 9

Find the image domain of the unit disk under the mapping $w = Arc\tan z$.

Prob. 10

Find the principal branch $Arc\tan z$ of the inverse of $\tan z$ (i.e. the branch whose restriction to the real axis coincides with the real function $Arc\tan z$ in terms of Log).