C. Methods not using derivatives

a. Uniform Search

Uniform search method is based on the evaluation of a function $\theta(\lambda)$ on small subintervals of $[a, b]$. These intervals have the same size δ. If the minimum of θ is located at some point λ^* then the minimum of θ is located in the subinterval $[\lambda^* - \delta, \lambda^* + \delta]$. This subinterval is then divided into many subintervals and the procedure is repeated.

b. Sequential Search Methods

A number of search methods are more efficient than the uniform search method. We here summarize some of these methods : Dichotmous search, Golden section method and Fibonacci method.

b.1 Dichotomous Search

This method is based on the assumption that $\theta(\lambda)$ is strictly quasi-convex on the interval $[a_1 , b_1]$.

Initialization :

Choose $2\varepsilon > 0$ and $l > 0$ the allowable final length of uncertainty.

Let $k=1$ and **go to main step**.

Main Step:

1. If $b_k - a_k < l$, Stop: the minimum is in the interval $[a_k, b_k]$. Else consider:

$$\lambda_k = (a_k + b_k)/2 - \varepsilon \qquad \mu_k = (a_k + b_k)/2 + \varepsilon,$$

go to step 2.

2. If $\theta(\lambda_k) < \theta(\mu_k)$, let $a_{k+1} = a_k$ and $b_{k+1} = \mu_k$. Else $a_{k+1} = \lambda_k$ and $b_{k+1} = b_k$. Replace k by $k+1$ and ***go to step 1***.

Hence we can determine the number of iterations required to achieve the desired accuracy.

b.2 Golden section method

This method is also based on the assumption that $\theta(\lambda)$ is strictly quasi-convex on the interval $[a_1 , b_1]$.

Initialization:

Choose $l > 0$ the allowable final length of uncertainty.

Let $\lambda_1 = a_1 + (1-\alpha)(b_1 - a_1)$ and $\mu_1 = a_1 + \alpha(b_1 - a_1)$, with $\alpha = 0.618$. Evaluate $\theta(\lambda_1)$ and $\theta(\mu_1)$, let $k=1$ and **go to main step.**

Main Step:

1. If b_k-a_k< l, stop : the minimum is in the interval $[a_k, b_k]$.
 Else if $\theta(\lambda_k) > \theta(\mu_k)$ **go to step 2**.
 If $\theta(\lambda_k) \leq \theta(\mu_k)$ **go to step 3**.

2. Let $a_{k+1}= \lambda_k$ and $b_{k+1}= b_k$. Let $\lambda_{k+1}= \mu_k$ and $\mu_{k+1}= a_{k+1}+ \alpha(b_{k+1}- a_{k+1})$, evaluate $\theta(\mu_{k+1})$ and **go to step 4**.
3. Let $a_{k+1}= a_k$ and $b_{k+1}= \mu_k$. Let $\lambda_{k+1}= a_{k+1}+ (1-\alpha)(b_{k+1}- a_{k+1})$ and $\mu_{k+1}= \lambda_k$, evaluate $\theta(\lambda_{k+1})$ and **go to step 4**.
4. Replace k by $k+1$ and **go to step 1.**

b.3 The Fibonacci Method

This method is also based on the assumption that $\theta(\lambda)$ is strictly quasi-convex on the interval $[a_1 , b_1]$.
The Fibonacci series $\{F_n\}$ is defined by:

$$F_{n+1} = F_n + F_{n-1}, \qquad n=1,2,\ldots$$
$$F_0= F_1=1$$

Initialization :
Choose l >0 the allowable final length of uncertainty and choose $\varepsilon > 0$.
Choose the number of observations n to perform $F_n > (b_1- a_1)/l$.
Let $\lambda_1 = a_1+(F_{n-2}/F_n)(b_1- a_1)$ and $\mu_1 = a_1+(F_{n-1}/F_n)((b_1- a_1)$. Evaluate $\theta(\lambda_1)$ and $\theta(\mu_1)$, let $k=1$ and go to main step.

Main Step :

1. If $\theta(\lambda_k) > \theta(\mu_k)$ go to step 2. If $\theta(\lambda_k) \leq \theta(\mu_k)$ go to step 3.
2. Let $a_{k+1}= \lambda_k$, $b_{k+1}= b_k$, $\lambda_{k+1}= \mu_k$ and $\mu_{k+1}= a_{k+1}+(F_{n-k-1}/F_{n-k})(b_{k+1}- a_{k+1})$.
 If $k=n-2$ go to step 5, else evaluate $\theta(\mu_{k+1})$ and go to step 4.

3. Let $a_{k+1}= a_k$, $b_{k+1}= \mu_k$, $\lambda_{k+1}= a_{k+1}+ (F_{n-k-2}/F_{n-k})(b_{k+1}- a_{k+1})$ and $\mu_{k+1}= \lambda_k$.
 If $k=n-2$ go to step 5, else evaluate $\theta(\lambda_{k+1})$ and go to step 4.

4. Replace k by $k+1$ and go to step 1.
5. Let $\lambda_n= \lambda_{n-1}$ and $\mu_n= \lambda_{n-1}+ \varepsilon$.
 If $\theta(\lambda_n) > \theta(\mu_n)$, Let $a_n= \lambda_n$ *and* $b_n= b_{n-1}$.
 Else $\theta(\lambda_n) \leq \theta(\mu_n)$, let $a_n= a_{n-1}$ *and* $b_n= \lambda_n$.

 Stop, the optimal solution is in the interval $[a_n , b_n]$.

Reference :
NonLinear Programming, Theory and Algorithms, Bazaraa M.S., Sherali H.D and Shetty C.M., second edition, WILEY.