King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
MATH-480 Linear and Nonlinear Programming [1]
Course Appendix I

Linearization of the complementary slackness conditions

In order to linearize the complementary slackness conditions corresponding to a linear program we develop the following procedure. Let (P) be the following linear program:

$$\begin{array}{ll} \max\limits_{x} & c^t x \\ \text{s.t} & Ax \leq b, \\ & x \geq 0. \end{array}$$

Where A is an $n \times m$ matrix.

Let also (D) be the dual corresponding to (P):

$$\begin{array}{ll} \min\limits_{\lambda} & \lambda^t b \\ \text{s.t} & \lambda^t A \geq c^t, \\ & \lambda \geq 0. \end{array}$$

The complementary slackness conditions corresponding to (P) and (D) can be expressed:

$$\lambda^t (Ax - b) = 0. \qquad (1)$$

These conditions can also be expressed as follows:

$$(\lambda^t A - c^t) x = 0. \qquad (2)$$

In order to linearize these conditions we introduce the binary vector of variables u and two real parameters K and L arbitrarily chosen with $K < 0$ and $L > 0$.

Conditions (1) can then be written as follows:

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\begin{aligned}
&Ax - b \geq Ku\\
&Ax - b \leq 0,\\
&\lambda \leq L(\mathbb{1} - u),\\
&\lambda^t A - c^t \geq 0,\\
&x \geq 0,\\
&\lambda \geq 0.
\end{aligned}
$$

Hence, for $i = 1, ..., m$ one can find that conditions (1) are satisfied if $u_i = 0$ or $u_i = 1$.

If $u_i = 0$ then $A_i x - b_i \geq 0$ and $A_i x - b_i \leq 0$.
Thus $A_i x - b_i = 0 \Rightarrow$ conditions (1) are satisfied.

If $u_i = 1$ then $A_i x - b_i \geq k$, $A_i x - b_i \leq 0$ and $\lambda_i \leq 0$.
Thus $\lambda_i = 0 \Rightarrow$ conditions (1) are satisfied.