

**King Fahd University of Petroleum and Minerals**  
**Department of Mathematics and Statistics**  
**MATH-480 Linear and Nonlinear Programming**<sup>1</sup>  
**Final Exam**

**Exercise 1** (30 points)

Consider the following linear program:

$$\begin{array}{ll} \min_{x_1, x_2, x_3} & 3x_1 + x_2 + 2x_3 \\ \text{s.t} & x_1 + x_2 + 2x_3 \leq 3, \\ & x_1 - x_2 + x_3 \geq 2, \\ & 2x_1 + x_2 + x_3 \leq 5, \\ & x_1, x_2, x_3 \geq 0. \end{array}$$

- (a) Solve the linear program using the Primal Simplex algorithm.
  
- (b) Solve the linear program using the Dual Simplex algorithm.

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<sup>1</sup>Dr. Slim Belhaiza (c), 29 January, 2010

**Exercise 2** (25 points)

Consider the following nonlinear program:

$$\min_{x,y} f(x,y) = x^2 + xy^2 + 2y^2$$

where  $x$  and  $y$  are real decision variables.

(a) Give the first order optimality conditions and find **all points** satisfying these conditions.

(b) Give the second order optimality conditions.

(c) Conclude on the optimality of each point found in (a).

**Exercise 3**(45 points)

Consider the following nonlinear program:

$$\min_{x_1, x_2} f(x_1, x_2) = (x_1 - 1)^4 + 2(x_1 - x_2)^2$$

where  $x_1$  and  $x_2$  are real decision variables.

- (a) Starting from the point  $(2, 2)$  perform 3 iterations of the Newton Method.
  
- (b) Adding the constraint:  $x_1 + x_2 - 2 = 0$ , Give the Kuhn-Tucker conditions corresponding to the constrained problem.
  
- (c) Perform 3 iterations of the Penalties Method to optimize the constrained problem.