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Department of Mathematics and Statistics

MATH 302-4

Quiz 2, Term 101

NAME: . . . .

ID : . . . .

Instructor: Prof.Dr. Othman Echi

Exercise. Let $A = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 2 & -2 \\ 2 & 0 & 1 \end{pmatrix}$.

(1) Show that A is invertible and find A^{-1}.

(2) Solve the system

$$A \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix}.$$

SOLUTION

1. Let us reduce the matrix $B = [A \vdots I_3] = \begin{pmatrix} 1 & 0 & -1 & \vdots & 1 & 0 & 0 \\ 0 & 2 & -2 & \vdots & 0 & 1 & 0 \\ 2 & 0 & 1 & \vdots & 0 & 0 & 1 \end{pmatrix}$.

We perform the following elementary row operations:

(i) $R_3 - 2R_1$ and $\frac{1}{2}R_2$ (on B) give the matrix

$$B_1 = \begin{pmatrix} 1 & 0 & -1 & \vdots & 1 & 0 & 0 \\ 0 & 1 & -1 & \vdots & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 3 & \vdots & -2 & 0 & 1 \end{pmatrix}$$

(ii) $\frac{1}{3}R_3$ (on B_1) gives the matrix

$$B_2 = \begin{pmatrix} 1 & 0 & -1 & \vdots & 1 & 0 & 0 \\ 0 & 1 & -1 & \vdots & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 & \vdots & \frac{-2}{3} & 0 & \frac{1}{3} \end{pmatrix}$$

(iii) $R_1 + R_3$ and $R_2 + R_3$(on B_2) give the matrix

$$B_3 = \begin{pmatrix} 1 & 0 & 0 & \vdots & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & 1 & 0 & \vdots & \frac{-2}{3} & \frac{1}{2} & \frac{1}{3} \\ 0 & 0 & 1 & \vdots & \frac{-2}{3} & 0 & \frac{1}{3} \end{pmatrix}$$

It follows that A is invertible and

$$A^{-1} = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} \\ -\frac{2}{3} & \frac{1}{2} & \frac{1}{3} \\ -\frac{2}{3} & 0 & \frac{1}{3} \end{pmatrix}.$$

2. The solution of the system is

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = A^{-1} \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}.$$