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Department of Mathematics and Statistics

MATH 302-4

Quiz 1, Term 101

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Exercise 1. Let $U = (1, 1, 0)$ and $V = (1, 1, \sqrt{2})$.

(1) Evaluate the angle between the two vectors U and V.
(2) Evaluate the area of the parallelogram defined by the vectors U and V.

Exercise 2. Let $U = (-1, 2, 1)$, $V = (2, 1, 1)$ and

$$S = \{W = (x, y, z) \in \mathbb{R}^3 \mid U \cdot W = 0 \text{ and } V \cdot W = 0\}.$$

(1) Show that S is a subspace of $\mathbb{R}^3$.
(2) Find a basis of S.

SOLUTIONS

Ex. 1. The angle between two nonzero vectors U, V is the real number $\theta \in [0, \pi]$ such that

$$\cos(\theta) = \frac{U \cdot V}{||U|| \times ||V||}\ .$$

The area of the parallelogram defined by the two vectors is

$$\mathcal{A}(U, V) = ||U|| \times ||V|| \sin(\theta)\ (\text{unit}^2)$$

In our case, $\cos(\theta) = \dfrac{1}{\sqrt{2}}$. Hence $\theta = \frac{\pi}{4}$ and consequently,

$$\mathcal{A}(U, V) = \sqrt{2} \times \sqrt{4} \times \frac{1}{\sqrt{2}} = 2\ (\text{unit}^2).$$

□

Ex. 2.

1.

– As $U \cdot 0_{\mathbb{R}^3} = V \cdot 0_{\mathbb{R}^3} = 0$, we get $0_{\mathbb{R}^3} \in S$.

– Let W_1, W_2 be two elements of S. Then

$$U \cdot W_1 = V \cdot W_1 = U \cdot W_2 = V \cdot W_2 = 0.$$

This leads to

$$U \cdot (W_1 + W_2) = U \cdot W_1 + U \cdot W_2 = 0 + 0 = 0,$$

and

$$V \cdot (W_1 + W_2) = V \cdot W_1 + V \cdot W_2 = 0 + 0 = 0.$$

It follows that $W_1 + W_2 \in S$.

– Now, let $W \in S$ and $\alpha \in \mathbb{R}$. Then

$$U \cdot (\alpha W) = \alpha U \cdot W = \alpha \times 0 = 0,$$

and

$$V \cdot (\alpha W) = \alpha V \cdot W = \alpha \times 0 = 0.$$

This implies that $\alpha W \in S$.

Therefore, S is a subspace of $\mathbb{R}^3$.

2. Let $W = (x, y, z) \in \mathbb{R}^3$. Then $W \in S$ if and only if the following equations are satisfied

$$-x + 2y - z = 0 \text{ and } 2x + y + z = 0.$$

This gives $y = 3x$ and $z = -5x$. Thus, $W = (x, y, z) = (x, 3x, -5x) = x(1, 3, -5)$. We conclude that $\mathcal{B} = \{(1, 3, -5)\}$ is a basis of S. □