Math 302

Quiz 4

20/ 12/ 2010

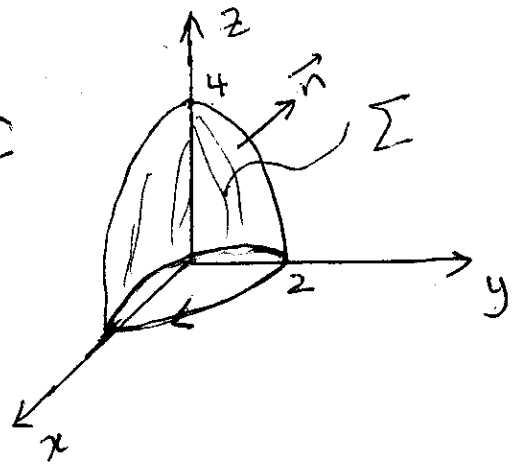
Name: **ID #**

Problem 1. (6 pts) Let Σ be surface above the xy-plane given by

$$\Sigma = \{(x, y, z) \in \mathbb{R}^3 \text{ such that } z = 4 - x^2 - y^2\}.$$

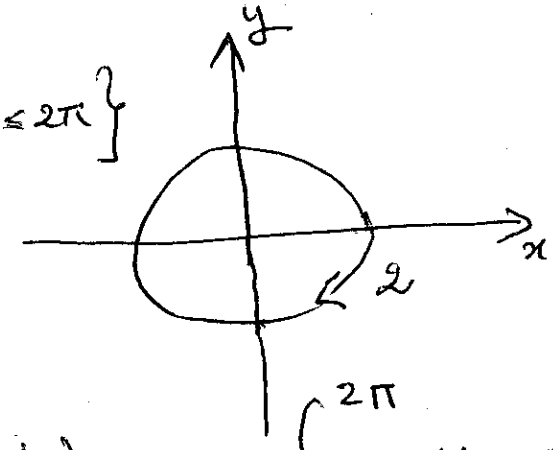
and the field $F(x, y, z) = -y\vec{i} + x\vec{j} + ze^{xy}\,\vec{k}$. Find $\int\int_\Sigma curl F \cdot n d\sigma$, where n is the unit normal pointing away of the surface.

Hint: you may think of Stokes' formula



Σ is limited by the circle C in xy plane centered at $(0,0,0)$ with radius $r=2$

Since $\vec{n}$ is pointing out then the circle is oriented clockwise.



$$-C = \{(2\cos t, 2\sin t, 0) \,/\, 0 \le t \le 2\pi\}$$

$$\iint_\Sigma curl F \cdot n \, d\sigma = \oint_C F \cdot dR$$

$$= -\int_0^{2\pi} \begin{pmatrix} -2\sin t \\ 2\cos t \\ 0 \end{pmatrix} \cdot \begin{pmatrix} -2\sin t \\ 2\cos t \\ 0 \end{pmatrix} dt = -\int_0^{2\pi} 4 \, dt = -8\pi.$$

Problem 2 (4pts): Find all the values z, for which

$$|z+2+i| > |z-1|$$

and graph it in the plane.

If $z = x+iy$ then

$$|(x+2)+i(y+1)| > |(x-1)+iy|$$

$$\Leftrightarrow \quad (x+2)^2 + (y+1)^2 > (x-1)^2 + y^2$$

$$\Leftrightarrow \quad x^2+2x+4+y^2+2y+1 > x^2-2x+1+y^2$$

$$4x+2y+4>0$$

$$2x+y+2>0$$

The set of value of z is

$$S = \{ z = x+iy \mid 2x+y+2>0 \}$$