Department of Math. and Stat.

Math 302 - Quiz 5

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Name: **ID #**

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Problem 1 (5 points):
Find all complex values z for which

$$f(z) = z + iz\,\mathrm{Im}\,z$$

is **not** differentiable

$f(z) = z + g(z) \;/\; g(z) = iz\,\mathrm{Im}\,z$

f is differentiable iff g is differentiable.

If $z = x+iy$ then $g(z) = i(x+iy)y = -y^2 + ixy$

$u = -y^2$ and $v = xy$

$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \iff 0 = x$

$\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y} \iff y = -2y \iff y = 0$

So g is not diffentiable at $z \neq 0$ (C-R do not hold at $z \neq 0$)

For $\boxed{z=0}$ $\lim_{h\to 0} \frac{g(0+h)-g(0)}{h} = \lim_{h\to 0} \frac{ih\,\mathrm{Im}\,h}{h}$

$= i \lim_{h\to 0} \mathrm{Im}\,h = 0 = g'(0)$

$\Rightarrow g$ is diff. at $z=0 \Rightarrow f$ is not diff at any $z \neq 0$ only.

Problem 2 (5 points):
Suppose that the series $\sum_{n=1}^{+\infty} c_n(z+i)^n$ converges at $z=1$.
Does converge or diverge at $z = 1-i$ (you have to justify your answer)

~~If~~ the series converges at $z=1 \Rightarrow |1+i| \leq R$

R is the radius of convergence

That is $R \geq \sqrt{2}$

For $z = 1-i$

$|1-i+i| = 1 < \sqrt{2} < R$

$\Rightarrow$ series converges at $z = 1-i$