Math 302

Quiz 2

23/ 10/ 2010

Name: **ID #**

Problem 1 (5 points): Find all the values of c, for which the system

$$\begin{aligned} 2x_1 \quad\;\; + x_3 &= 10 \\ 3x_1 + x_2 - x_3 &= 5 \\ -x_1 - x_2 + 2x_3 &= c \end{aligned}$$

has no solution.

$$\left(\begin{array}{ccc|c} 2 & 0 & 1 & 10 \\ 3 & 1 & -1 & 5 \\ -1 & -1 & 2 & c \end{array}\right) \xrightarrow{R_1 \leftrightarrow -R_3} \left(\begin{array}{ccc|c} 1 & 1 & -2 & -c \\ 3 & 1 & -1 & 5 \\ 2 & 0 & 1 & 10 \end{array}\right) \xrightarrow[R_3 - 2R_1]{R_2 - 3R_1}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -2 & c \\ 0 & -2 & 5 & 5+3c \\ 0 & -2 & 5 & 10+2c \end{array}\right) \xrightarrow[R_3 - R_2]{\substack{R_1 \\ -R_2/2}} \left(\begin{array}{ccc|c} 1 & 1 & -2 & c \\ 0 & 1 & -\frac{5}{2} & \frac{-5-3c}{2} \\ 0 & 0 & 0 & 5-c \end{array}\right)$$

It is clear that the system has no solution iff $5 - c \neq 0$ iff $c \neq 5$.

Problem 2 (5points):
Find all the eigenvalues of the matrix

$$A = \begin{pmatrix} 2 & 0 & 1 \\ 3 & 1 & -1 \\ -1 & -1 & 2 \end{pmatrix}$$

$$\det(A - \lambda I) = \begin{vmatrix} 2-\lambda & 0 & 1 \\ 3 & 1-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix}$$

$$= (2-\lambda)\begin{vmatrix} 1-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} + 1\begin{vmatrix} 3 & 1-\lambda \\ -1 & -1 \end{vmatrix}$$

$$= (2-\lambda)(\lambda^2 - 3\lambda + 1) + (-2-\lambda)$$

$$= 2\lambda^2 - 6\lambda + \cancel{2} - \lambda^3 + 3\lambda^2 - \lambda - \cancel{2} - \lambda$$

$$= -\lambda^3 + 5\lambda^2 - 8\lambda = -\lambda(\lambda^2 - 5\lambda + 8) = 0$$

$$\Longrightarrow \quad \lambda = 0 \quad \text{or} \quad \lambda^2 - 5\lambda + 8 = 0 \Longrightarrow$$

$$\lambda = \frac{5 \pm i\sqrt{7}}{2}.$$

So the eigenvalues are

$$\lambda = 0, \ \frac{5 \pm i\sqrt{5}}{2}.$$