King Fahd University of Petroleum and Minerals

Department of Mathematics and Statistics

Semester (101)   January 26, 2011   Time: 07:00 - 09:30 pm

Name: ....

ID: ....   Section: ...

Problem	Points
1	____ / 5
2	____ / 5
3	____ / 5
4	____ / 5
5	____ / 5
6	____ / 5
7	____ / 5
Total	____ / 35

Problem 1. Consider the matrix

$$A = \begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}.$$

(a) Find the eigenvalues of A.

(b) Find a nonsingular matrix P that diagonalizes A.

Problem 2. Given a vector field

$$F(x, y, z) = \frac{y}{1 + xy}\mathbf{i} + \frac{x}{1 + xy}\mathbf{j} + \frac{1}{z}\mathbf{k},$$

for $x \geq 0, y \geq 0$ and $z > 0$.

(a) Show that F is conservative.

(b) Find a potential ϕ of F.

(c) Evaluate the integral $\int_\Gamma F.dR$, where Γ is the line segment joining the point $A = (0, 0, 1)$ and $B = (1, 1, 2)$.

Problem 3. Given the cone $\Sigma = \{(x, y, z) : 0 \leq z = 2 - \sqrt{x^2 + y^2}\}$ and the vector field $F(x, y, z) = y\mathbf{i} + xy\mathbf{j} + e^{-z^2}\mathbf{k}$. Evaluate the integral

$$\int\int_{\Gamma} \mathrm{Curl}(F).\mathbf{n} d\sigma,$$

where $\mathbf{n}$ is the unit outer normal to Σ.

Problem 4. Let $f : \mathbb{C} \longrightarrow \mathbb{C}$ be a differentiable complex function such that $\mathrm{Re}(f(z)) = x^2 - y^2$, for $z = x + iy$. Use Cauchy Riemann Equations to find $\mathrm{Im}(f(z))$.

Problem 5. Use Cauchy integral Formula to evaluate the integral

$$\oint_{\Gamma} \frac{1+\cos(iz)}{z(z+1)^2} dz,$$

where Γ is the positively oriented circle given by $|z+1| = \frac{1}{2}$.

Problem 6. Find Laurent series of the function $f(z) = z^5 \sin(\frac{1}{z^2})$ about 0 and use it to compute the integral

$$\oint_C f(z)dz,$$

where C is the positively oriented circle of center 0 and radius $r > 0$.

Problem 7. Let $f(z) = \dfrac{z^2 + z + 1}{\sin z}$ and Γ be the positively oriented circle of center 0 and radius 4.

Use Residue Theorem to evaluate the integral $\oint_\Gamma f(z)dz$.