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Department of Mathematics and Statistics

MATH 302-2

Quiz 2, Term 101

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Exercise. Let $A = \begin{pmatrix} 1 & 1 & 2 \\ 2 & 1 & 2 \\ 1 & 1 & 1 \end{pmatrix}$.

(1) Show that A is invertible and find A^{-1}.

(2) Solve the system

$$A \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}.$$

SOLUTION

1. Let us reduce the matrix $B = [A \vdots I_3] = \begin{pmatrix} 1 & 1 & 2 & \vdots & 1 & 0 & 0 \\ 2 & 1 & 2 & \vdots & 0 & 1 & 0 \\ 1 & 1 & 1 & \vdots & 0 & 0 & 1 \end{pmatrix}$.

We perform the following elementary row operations:

(*i*) $R_2 - R_1$ and $R_3 - R_1$ (on B) give the matrix

$$B_1 = \begin{pmatrix} 1 & 1 & 2 & \vdots & 1 & 0 & 0 \\ 0 & -1 & -2 & \vdots & -2 & 1 & 0 \\ 0 & 0 & -1 & \vdots & -1 & 0 & 1 \end{pmatrix}$$

(*ii*) $(-1)R_2$ and $(-1)R_2$ (on B_1) give the matrix

$$B_2 = \begin{pmatrix} 1 & 1 & 2 & \vdots & 1 & 0 & 0 \\ 0 & 1 & 2 & \vdots & 2 & -1 & 0 \\ 0 & 0 & 1 & \vdots & 1 & 0 & -1 \end{pmatrix}$$

(*iii*) $R_1 - R_2$ (on B_2) gives the matrix

$$B_3 = \begin{pmatrix} 1 & 0 & 0 & \vdots & -1 & 1 & 0 \\ 0 & 1 & 2 & \vdots & 2 & -1 & 0 \\ 0 & 0 & 1 & \vdots & 1 & 0 & -1 \end{pmatrix}$$

(*iv*) $R_2 - 2R_3$ (on B_3) gives the matrix

$$B_4 = \begin{pmatrix} 1 & 0 & 0 & \vdots & -1 & 1 & 0 \\ 0 & 1 & 0 & \vdots & 0 & -1 & 2 \\ 0 & 0 & 1 & \vdots & 1 & 0 & -1 \end{pmatrix}$$

It follows that A is invertible and

$$A^{-1} = \begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 0 & -1 \end{pmatrix}.$$

2. The solution of the system is

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = A^{-1} \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}.$$