**King Fahd University of Petroleum and Minerals**
**Department of Mathematics and Statistics**

# FINAL EXAM MATH 302

Semester (101) January 26, 2011 Time: 07:00 - 09:30 pm

Name: ....................................

ID: ............... Section: ...

| Problem | Points |
|---|---|
| 1 | ____ / 5 |
| 2 | ____ / 5 |
| 3 | ____ / 5 |
| 4 | ____ / 5 |
| 5 | ____ / 5 |
| 6 | ____ / 5 |
| 7 | ____ / 5 |
| Total | ____ / 35 |

**Problem 1.** Consider the matrix

$$A = \begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}.$$

($a$) Find the eigenvalues of $A$.

($b$) Find a nonsingular matrix $P$ that diagonalizes $A$.

**Problem 2.** Given a vector field

$$F(x, y, z) = \frac{y}{1+xy}\mathbf{i} + \frac{x}{1+xy}\mathbf{j} + \frac{1}{z}\mathbf{k},$$

for $x \geq 0, y \geq 0$ and $z > 0$.

($a$) Show that $F$ is conservative.

($b$) Find a potential $\phi$ of $F$.

($c$) Evaluate the integral $\int_\Gamma F.dR$, where $\Gamma$ is the line segment joining the point $A = (0, 0, 1)$ and $B = (1, 1, 2)$.

**Problem 3.** Given the cone $\Sigma = \{(x, y, z) : 0 \leq z = 2 - \sqrt{x^2 + y^2}\}$ and the vector field $F(x, y, z) = y\mathbf{i} + xy\mathbf{j} + e^{-z^2}\mathbf{k}$. Evaluate the integral

$$\int\int_{\Sigma} \mathrm{Curl}(F).\mathbf{n} d\sigma,$$

where $\mathbf{n}$ is the unit outer normal to $\Sigma$.

**Problem 4.** Let $f : \mathbb{C} \longrightarrow \mathbb{C}$ be a differentiable complex function such that $\mathrm{Re}(f(z)) = x^2 - y^2$, for $z = x + iy$. Use Cauchy Riemann Equations to find $\mathrm{Im}(f(z))$.

**Problem 5.** Use Cauchy integral Formula to evaluate the integral

$$\oint_\Gamma \frac{1+\cos(iz)}{z(z+1)^2}dz,$$

where $\Gamma$ is the positively oriented circle given by $|z+1| = \frac{1}{2}$.

**Problem 6.** Find Laurent series of the function $f(z) = z^5 \sin(\frac{1}{z^2})$ about 0 and use it to compute the integral

$$\oint_C f(z)dz,$$

where $C$ is the positively oriented circle of center 0 and radius $r > 0$.

**Problem 7.** Let $f(z) = \dfrac{z^2 + z + 1}{\sin z}$ and $\Gamma$ be the positively oriented circle of center 0 and radius 4.

Use Residue Theorem to evaluate the integral $\oint_\Gamma f(z)dz$.