King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 302 Exam II

Semester (101) December 9, 2010 Time: 12:30 - 14:15 pm

Name: .... I.D: .... Section: ...

Problem	Points
1	___ / 8
2	___ / 8
3	___ / 12
4	___ / 10
5	___ / 12
Total	___ / 50

Problem 1.

(a) Let φ be the scalar field defined by $\varphi(x, y, z) = x^2yz$. Compute $\mathrm{grad}(\varphi)$ and $\mathrm{curl}(\mathrm{grad}(\varphi))$.

(b) Let F be the vector field defined by

$$F(x, y, z) = (y + e^y, z + e^z, x + e^x).$$

Is there a scalar field ψ with continuous first and second partial derivatives such that $F = \mathrm{grad}(\psi)$?

Problem 2. Find the length of the curve C given by

$$y = 2x^2 - \frac{1}{16}\ln x, \quad 1 \leq x \leq 3.$$

Problem 3. Let $C = C_1 \cup C_2 \cup C_3$ be the positively oriented closed path in $\mathbb{R}^2$, with 3 smooth pieces:

- C_1 : the straight line segment joining $(0,0)$ and $(1,0)$.
- C_2 : the quarter circle (of center the origin) joining $(1,0)$ and $(0,1)$.
- C_3 : the straight line segment joining $(0,1)$ and $(0,0)$.

A vector field is given by $F(x,y) = (x+y)\mathbf{i} + y\mathbf{j}$.

Verify Green's theorem, by computing the line and the double integrals.

Problem 4. Let Σ be the surface of $\mathbb{R}^3$ given by $z = x + 2y^2$, with $0 \leq x \leq 1$ and $0 \leq y \leq \sqrt{6}$. Evaluate the surface integral

$$I = \int\int_{\Sigma} y d\sigma.$$

Problem 5. Let Σ be the closed surface which consists of the part of the cylinder $z^2 + x^2 = 4$ lying in the first octant and the parts of the planes $y = 0$, $y = 3$, $z = 0$, $x = 0$, as shown in the figure

Given a field F by $F(x, y, z) = xz^2\mathbf{i} + (x^2 - z + y)\mathbf{j} + zx^2\mathbf{k}$. Evaluate the flux of F across Σ

$$Q = \int\int_{\Sigma} F.n \; d\sigma.$$