King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 301 Final Exam
The First Semester of 2010-2011 (101)

Time Allowed: 150 Minutes

Name: ______________________ ID#: ____________

Instructor: ____________________ Sec #: ______ Serial #: ______

- Mobiles and calculators are not allowed in this exam.
- Write all steps clear.

Question #	Marks	Maximum Marks
1		10
2		12
3		16
4		20
5		20
6		15
7		15
Total		108

Q:1 (a) (5 points) Find Laplace transform $\mathcal{L}\{f(t)\}$ where

$$f(t) = \begin{cases} 0 & \text{if } 0 \le t < 1 \\ t^2 & \text{if } \quad t \ge 1 \end{cases}.$$

(b) (5 points) Find inverse Laplace transform $\mathcal{L}^{-1}\left\{\dfrac{e^{-2s}}{s^2(s-1)}\right\}$.

Q:2 (12 points) Let $\vec{F}(x, y, z) = \dfrac{x\mathbf{i} + y\mathbf{j} + z\mathbf{k}}{x^2 + y^2 + z^2}$ and D is the region bounded by the concentric spheres $x^2+y^2+z^2 = 9$ and $x^2+y^2+z^2 = 4$. Use divergence theorem to evaluate $\iint_S \left(\vec{F} \cdot \hat{n}\right) dS$.

Q:3 (16 points) Use separation of variables method to find the nontrivial solution of the heat equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \quad 0 < x < 1, \quad t > 0$$

subject to the boundary and initial conditions

$$\begin{aligned} u(0,t) &= 0, \quad u(1,t) = 0, \quad t > 0 \\ u(x,0) &= 10, \quad 0 < x < 1. \end{aligned}$$

Q:4 (20 points) Use separation of variables method to find the nontrivial solution of the wave equation

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} = \frac{\partial^2 u}{\partial t^2}, \ 0 < r < 2, \ t > 0$$

subject to the boundary conditions

$$\begin{aligned} u(2,t) &= 0, \ t > 0 \\ u(r,0) &= 1, \ \left.\frac{\partial u}{\partial t}\right|_{t=0} = 0, \quad 0 < r < 2 \\ \text{solution is bounded at } r &= 0. \end{aligned}$$

Q:5 (20 points) Find the steady-state temperature $u(r,\theta)$ in a sphere of radius 2 by solving the problem

$$\frac{\partial^2 u}{\partial r^2} + \frac{2}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} + \frac{\cot\theta}{r^2}\frac{\partial u}{\partial \theta} = 0, \quad 0 < r < 2,\ 0 < \theta < \pi,$$

subject to the boundary

$$u(2,\theta) = 1 + \cos(\theta), \quad 0 < \theta < \pi.$$

Q:6 (15 points) Use Laplace transform to solve the problem

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, \quad 0 < x < L, \ t > 0$$

subject to the boundary and initial conditions

$$\begin{aligned} u(0,t) &= 0, \quad u(L,t) = 0, \quad t > 0 \\ u(x,0) &= 2\sin(\frac{\pi x}{L}), \quad \left.\frac{\partial u}{\partial t}\right|_{t=0} = 0, \quad 0 < x < L. \end{aligned}$$

Q:7 (15 points) Use appropriate Fourier transform to solve the problem

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \quad x > 0, \ t > 0$$

subject to the conditions

$$\begin{aligned} u(0,t) &= 5, t > 0 \\ u(x,0) &= 0, \quad x > 0. \end{aligned}$$