King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 301 Major Exam 2
The First Semester of 2010-2011 (101)
Time Allowed: 120 Minutes

Name: _____ ID#: _____

Instructor: _____ Sec #: _____ Serial #: _____

- Mobiles and calculators are not allowed in this exam.
- Write all steps clear.

Question #	Marks	Maximum Marks
1		14
2		20
3		16
4		14
5		14
6		10
Total		88

Q:1 (a) (14 points) Use Laplace transform to solve the initial value problem

$$y'' - y' = \cos(t),\ y(0) = 0,\ y'(0) = -1.$$

Q:2 (a) (6 points) Find Laplace transform $\mathcal{L}\{t\cos(2t)\}$.

(b) (6 points) Find Laplace transform $\mathcal{L}\{e^t\sin(2t)\cos(2t)\}$.

(c) (8 points) Find inverse Laplace transform $\mathcal{L}^{-1}\left\{\dfrac{1}{(s^2+9)^2}\right\}$.

Q:3 (a) (8 points) Use Laplace transform to solve the Volterra integral equation

$$f(t) = 3t^2 - \int_0^t f(\tau)e^{t-\tau}d\tau.$$

(b) (8 points) Use Laplace transform to solve the initial value problem

$$y'' + 4y' + 5y = \delta(t - 2\pi),\ y(0) = 0,\ y'(0) = 0.$$

Q:4 (14 points) Show that the set of functions $\{\cos(2n+1)x\}$ is an orthogonal set on $\left[0, \frac{\pi}{2}\right]$ for $n = 0, 1, 2, 3, \cdots$. Also find norm of each function. (Justify your answer with reason).

Q:5 (14 points) Find Fourier series expansion of

$$f(x) = \begin{cases} 0 & \text{if } -3 < x < 0 \\ 3 - x & \text{if } 0 \leq x < 3 \end{cases}.$$

Q:6 (a) (10 points) Find half range cosine expansion of

$$f(x) = x + 1, \quad 0 < x < \pi.$$