King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 301 Major Exam I
The Summer Semester of 2010-2011 (101)

Time Allowed: 120 Minutes

Name: ______________________ ID#: ______________

Section/Instructor: ______________ Serial #: ______________

- Mobiles and calculators are not allowed in this exam.
- Write all steps clear.

Question #	Marks	Maximum Marks
1		10
2		13
3		13
4		12
5		14
6		14
7		12
Total		88

Q:1 (a) (10 points) Evaluate the integral $\int_C x(x+y^2)dx + ydy$, along the curve C given by $x=\sqrt{2t}, y=t,\ 1\le t\le 2$.

Q:2 (a) (7 points) Find the directional derivative of $f(x, y, z) = 2xz + 3xy^2 + yz^2$ at $(-1, 1, 2)$ in the direction of $2\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}$.

(b) (6 points) Write the direction of maximum directional derivative and value of maximum directional derivative.

Q:3 (13 points) Determine whether the vector field $\vec{F}(x, y, z) = (2x \sin y + e^{3z})\,\mathbf{i} + (x^2 \cos y)\,\mathbf{j} + (3xe^{3z} + 5)\,\mathbf{k}$ is a gradient field. If so, find the potential function $\phi(x, y, z)$ for $\vec{F}$.

Q:4 (12 points) Use Green's Theorem to evaluate the integral $\oint_C 3e^{-x^2}\,dx + 2\tan^{-1} x\; dy$, where C is the positively oriented triangle with vertices $(0,0)$, $(0,2)$, $(-2,2)$.

Q:5 (14 points) Find the surface area of the portions of the sphere $x^2 + y^2 + z^2 = 25$ that are within the cylinder $x^2 + y^2 = 5y$.

Q:6 (14 points) Let $\vec{F}(x,y,z) = y^3\mathbf{i} - \mathbf{x}^3\mathbf{j} + z^3\mathbf{k}$. Use Stokes' theorem to evaluate the integral $\oint_C \vec{F} \cdot d\vec{r}$, where C is the trace of the cylinder $x^2 + y^2 = 1$ in the plane $x + y + z = 1$.

Q:7 (12 points) Let $\vec{F}(x, y, z) = y^2 z\mathbf{i} + x^3 z^2 \mathbf{j} + (z+2)^2 \mathbf{k}$ and D is the region bounded by the cylinder $x^2 + y^2 = 9$ and the planes $z = 1,\ z = 4$. Use divergence theorem to evaluate $\int\int_S \left(\vec{F} \cdot \hat{n}\right) ds$.