KFUPM – Department of Mathematics and Statistics – Term 101

MATH 280

Final Exam (January 24, 2011)

Duration: 3 Hours

NAME:________________________ ID:________________________ Section: _01_

Exercise 1 (20 points)

Use the augmented matrix to find all values of r for which the system (S) has:

a/ No solution b/ a unique solution c/ infinitely many solutions

$$(S)\ \begin{cases} x + 2y + z = 4 \\ x - 2y + r^2 z = r \\ x + 6y + z = 7 \end{cases}$$

Exercise 2 (20 points)

1-Which one of the following transformation is linear?

$(i) L_1 : R_3 \to R_3, L_1(a,b,c) = (a,b,c+1), (ii) L_2 : R_3 \to R_3, L_1(a,b,c) = (a-b, b-c, c-a)$

2-Find $KerL_2$, and $\dim KerL_2$

3-Find $RangeL_2$ and $\dim RangeL_2$

Exercise 3 (20 points)

Let P_3 be the vector space of all real polynomials of degree ≤ 3, P_2 be the vector space of all real polynomials of degree ≤ 2 and $D: P_3 \to P_2, D(f) = f'$ be the differential operator. Let $S = \{1, t, t^2, t^3\}$ and $T = \{1, t, t^2\}$ be the standard bases of P_3 and P_2 respectively. Set $S' = \{2, 1-t, -t^2, t^2 - t^3\}$ and $T' = \{1, 1-t, 1-t^2\}$.

1-Find the transition matrix P from S' to S.

2-Find the transition matrix Q from T' to T.

3-Find the matrix representing D with respect to S' and T'

Exercise 4 (20 points)

Use Gram-Schmidt process to transform the basis $S = \{\begin{pmatrix}1\\2\\2\end{pmatrix}, \begin{pmatrix}1\\0\\1\end{pmatrix}, \begin{pmatrix}1\\1\\1\end{pmatrix}\}$ to an orthonormal basis.

Exercise 5 (20 points)

Let V and W be two vector spaces over the same field k and suppose that $\dim V = \dim W$ is finite.

1-Prove that V and W are isomorphic.

2-Deduce that every real vector space V of dimension n is isomorphic to R^n.

Exercise 6 (20 points)

1-Use the properties of determinant to set-up a formula for the area of the rectangle (Δ) with vertices (a_1, b_1), (a_2, b_2) and (a_3, b_3).

2-Application: Find the area of the triangle with vertices $(1,2)$, $(2,4)$ and $(3,1)$.

Exercise 7 (20 points)

Let A be an $n \times n$ matrix with characteristic polynomial $f = X^n + a_{n-1}X^{n-1} + ... + a_1 X + a_0$ and suppose that $a_0 \neq 0$.

1-Prove that A is invertible and find A^{-1}. (Hint, use Cayley-Hamilton Theorem).

2-Application: Find the 3×3 matrix A such that $f = -X^3 + 2X^2 - 1$ and

$$2A - A^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix}$$

Exercise 8 (20 points)

Let $A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$. 1-Prove that A is diagonalizable.

2-Find an orthogonal matrix P such that $P^{-1}AP = D$ is diagonal.

Exercise 9 (20 points)
Let A be a real symmetric matrix.
1-Prove that the eigenvalues of A are all real numbers.
2-Prove that eigenvectors associated to distinct eigenvalues are orthogonal.

Exercise 10 (20 points)

Let $g(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz + 2yz$ be a quadratic form of R_3.

1-Find the canonical quadratic form h that is equivalent to g.

2-Find the rank and the signature of g