KFUPM – Department of Mathematics and Statistics – Term 101

MATH 280

Exam 2 (Due, November 24, 2010)

NAME:________________________ ID:________________________ Section: _01_

Exercise 1 (10 points)

Let (S) be the nonhomogeneous system given by $AX = Y$ where A is an mxn matrix. Under which condition a linear combination of r solutions $X_1, X_2, ..., X_r$ is a solution of the system (S).

Exercise 2 (15 points)

Let $N = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ and $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

1-Find N^2, N^3.

2-Use the expression $A = I + N$ to find A^3.

3-Use the expression of A^3 to find A^{-1}. **<u>No other method is accepted</u>**

Exercise 3 (10 points)
Find a $3x3$ nonsingular matrices A and B such that $A+B$ is a **nonzero** singular matrix.

Exercise 4 (10 points)

Let $A = \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix}$ be a partitioned matrix.

1-Prove that if A_{11} and A_{22} are nonsingular, then A is nonsingular.

2-Find an expression of A^{-1}.

Exercise 5 (15 points)
Let V be a vector space over a field K and $\Phi : V \to V$ a homomorphism.
Let $Ker\Phi = \{x \in V \mid \Phi(x) = 0\}$ and $\operatorname{Im}\Phi = \{\Phi(x) \mid x \in V\}$.
1-Prove that $Ker\Phi$ and $\operatorname{Im}\Phi$ are subspaces of V.
2-Suppose that $\Phi = \Phi^2$ (equivalent to $\Phi = \Phi o \Phi$). Prove that $Ker\Phi \cap \operatorname{Im}\Phi = \{0\}$
3-Prove that every element x of V can be uniquely expressed as $x = y + z$
where $y \in Ker\Phi$ and $z \in \operatorname{Im}\Phi$

Exercise 6 (10 points)
Let V be the vector space of all real-valued continuous functions and W be the subspace of V spanned by the functions 1, $\cos x$, and $\sin x$. Find a basis and the dimension of W.

Exercise 7 (15 points)

Let P_2 be the vector space of all polynomials of $K[t]$ of degree ≤ 2. Let $S = \{t, 1+t, t^2\}$ and $T = \{2+t, t, 1+t^2\}$. Find the transition matrix from T to S

Exercise 8 (15 points)
Let $V = \mathrm{M}_{nxn}(R)$ be the vector space of all nxn matrices, P be a nonsingular nxn matrix and $\Phi : V \to V, \Phi(A) = P^{-1}AP$.
Prove that Φ is an isomorphism.