KFUPM – Department of Mathematics and Statistics – Term 101

MATH 280

Exam 1 (Duration = 90 minutes)

NAME:________________________ ID:________________________ Section: _01_

Exercise 1 (15 points)

Use the augmented matrix to find all values of r for which the system (S) has:

a/ No solution b/ a unique solution c/ infinitely many solutions

$$(S)\ \begin{cases} x + y - z = 3 \\ x + y + rz = r \\ x - y - z = 2 \end{cases}$$

Exercise 2 (20 points)

Let $N = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ and $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

1-Find N^2, N^3 and N^n for all $n \geq 3$.

2-Find A^n fo all $n \geq 3$. [Hint: remark that $A = I + N$]

3-Use question 2- to find A^{100}

Exercise 3 (15 points)
Use elementary row operations to find the inverse of the matrix

$$A = \begin{pmatrix} 1 & 1 & -1 \\ 1 & 1 & 1 \\ 2 & -1 & 1 \end{pmatrix}$$

Exercise 4 (20 points)
Find all 3x3 matrices A such that $AB = BA$ for every 3x3 matrix B

Exercise 5 (15 points)
Determine whether W is a subspace of the given space V.
1-$V = K[X]$, the vector space of polynomials with coefficients in K, and
$W = \{ f \in K[X] \| f(3) = 0\}$

2-$V = R_3$ and $W = \{(a,b,c) \in R_3 | 2a - b + 2c = 0\}$

3-$V = R_3$ and $W = \{(a,b,c) \in R_3 | a - b + e^c = 1\}$

Exercise 6 (15 points)
Let A be an nxn symmetric matrix and B an nxn skew symmetric matrix.
1-Find all values of α such that $A+(\alpha^2-1)B$ is symmetric.

2-Under which condition AB is symmetric?

3- Find a 2x2 symmetric matrix A and a 2x2 skew symmetric matrix B such that $AB=-BA$