Name: ID ♯ : section:

Exercise1: (06 pts)

Consider the second order differential equation

(1)

$$y'' + y' = 1.$$

Let $y_1(x) = 1$ be a solution of the associated homogenous equation of (1).

1)-Use the method of reduction of order to find a second solution $y_2(x)$.

2)- Find the general solution of (1).

solution:

1) Let $y = u(x)y_1(x) = u(x)$ (01), Then $y' = u'$ and $y'' = u''$ and (01)

$y'' + y' = 0$ implies $u'' + u' = 0 \xRightarrow{w=u'} w' + w = 0 \Rightarrow \frac{dw}{dx} = -w$

$\Rightarrow \frac{dw}{w} = -dx$ Then $w = c_1 e^{-x} = u' \Rightarrow u = -c_2 e^{-x}$

(choose $c_2 = 1$) $\Rightarrow u = -e^{-x}$ and $\boxed{y_2 = -e^{-x}}$ (01)

So $\boxed{y_c = c_1 + c_2 e^{-x}}$ (01); clear That $\boxed{y_p = x}$ (01) is a particular

Solution of (1) and Hence The General Solution of (1) is

given by: $\boxed{y = y_c + y_p = c_1 + c_2 e^{-x} + x}$ (01)

Exercise2:

Solve $y''' - 4y'' - 5y' = 0$. (04 pts)

solution:

The Auxiliary equation is $m^3 - 4m^2 - 5m = 0$ (01) $\Rightarrow$

$m(m^2 - 4m - 5) = 0 \Rightarrow m(m-5)(m+1) = 0$ (01) $\Rightarrow m_1 = 0, m_2 = 5, m_3 = -1$ (01)

Hence The General Solution is given by:

$$\boxed{y = c_1 + c_2 e^{5x} + c_3 e^{-x}}$$ (01)

Exercise3: (07pt)

Using the method of undetermined Coefficients solve The DE: $y'' + 25y = 6\sin x$

solution:

We have $m^2+25=0 \Rightarrow m=\pm 5i$ So $y_c = c_1\cos 5x + c_2 \sin 5x$ (02)

or $(D^2+1)\sin x = 0 \Rightarrow (D^2+1)(6\sin x)=0$, Then

$(D^2+1)(D^2+25)y = (D^2+1)(6\sin x) = 0 \Rightarrow (m^2+1)(m^2+25)=0$

$\Rightarrow m_1 = 5i,\ m_2=-5i,\ m_3 = i$ and $m_4=-i$, So The general solu: is: (02)

$y = \underbrace{c_1\cos 5x + c_2\sin 5x}_{y_c} + c_3\cos x + c_4 \sin x$, Let $y_p = A\cos x + B\sin x$ (01)

after Substituting: (01) $24(A\cos x + B\sin x) = 6\sin x \Rightarrow \begin{cases} 24A=0 \\ 24B=6 \end{cases} \Rightarrow \begin{cases} A=0 \\ B=\frac{1}{4} \end{cases}$ (01)

Hence The General Solution is: $\boxed{y = c_1\cos 5x + c_2 \sin 5x + \frac{1}{4}\sin x}$

Exercise4: (03pt)

Solve the Cauchy-Euler equation

$$3x^2y'' + 6xy' + y = 0$$

solution:

Let $y = x^m$, after substitution we get $x^m[3m^2+3m+1]=0$ (01)

Then $m = \frac{-3\pm i\sqrt{3}}{6} = -\frac{1}{2} \pm i\frac{\sqrt{3}}{6}$, Hence The General

Solution is given by: $\boxed{y = x^{-\frac{1}{2}}\left[c_1\cos\left(\frac{\sqrt{3}}{6}\ln x\right) + c_2 \sin\left(\frac{\sqrt{3}}{6}\ln x\right)\right]}$ (02)