October 30,2010

QUIZ♯2 Math202, sec 08

Net Time Allowed: 20 minutes

Name: **ID ♯:** **section:**

Exercise1:

Consider the differential equation: $y(y^2\cos x+1)\,dx+(y^2\sin x-x+y)\,dy=0.$ (1)

(a) Is the DE (1) exact? Justify your answer !

(b) Find an integrating factor which makes the DE (1) exact.

(c) Solve the DE (1).

solution:

a/ Let $M(x,y)=y(y^2\cos x+1)$ and $N(x,y)=y^2\sin x-x+y$

Since $\frac{\partial M}{\partial y}=3y^2\cos x+1$ and $\frac{\partial N}{\partial x}=y^2\cos x-1$, (DE) (1) is not exact

b/ $\frac{N_x-M_y}{M}=-\frac{2}{y}$ which depend only of y. Hence an integrating factor is given by: $e^{-\int\frac{2}{y}dy}=e^{-\ln y^2}=\frac{1}{y^2}$.

c/ Multiplying the (DE) (1) by the integrating factor, gives:

$\left(y\cos x+\frac{1}{y}\right)dx+\left(\sin x-\frac{x}{y^2}+\frac{1}{y}\right)dy=0$, which is exact!

Let $f(x,y)$ be a function such that: $\frac{\partial f}{\partial x}=y\cos x+\frac{1}{y}$ (2) and $\frac{\partial f}{\partial y}=\sin x-\frac{1}{y^2}+\frac{1}{y}$ (3)

• Differentiate (3) with respect to y gives:

$\frac{\partial f}{\partial y}=\sin x-\frac{x}{y^2}+g'(y)=\sin x-\frac{x}{y^2}+\frac{1}{y}$. Thus $g'(y)=\frac{1}{y}$.

and So $g(y)=\ln|y|+c$, Therefore $y\sin x+\frac{x}{y}+\ln|y|=C$ is an implicit Solution of the (DE) (1).

Exercise2:

Solve the DE:

Solution of Exercise 2: (DE): $(\cot x - y^2 \sin^3 x)\,dx + \frac{1}{y}\,dy = 0$

This is a Bernoulli equation with $n = 3$.

Let $u = y^{1-3} = y^{-2} \Rightarrow y^2 = u^{-1} \Rightarrow y = u^{-\frac{1}{2}}$

So $\frac{dy}{dx} = -\frac{1}{2} u^{-\frac{3}{2}} \frac{du}{dx}$

$\Rightarrow -\frac{1}{2} u^{-\frac{3}{2}} \frac{du}{dx} + (\cot x)\, u^{-\frac{1}{2}} = u^{-\frac{3}{2}} \sin^3 x$

Hence

$\frac{du}{dx} - 2(\cot x)u = -2\sin^3 x$ which is a linear equation.