

$$\Rightarrow C_{k+2} = \frac{k+1}{k+2} C_{k+1} \quad | \quad k=1, 2, \dots$$

\* We choose  $C_0 = 1$  and  $C_1 = 0$  :

$$\text{So } C_2 = \frac{1}{2} \cdot 0 = 0$$

$$k=1 \Rightarrow C_3 = \frac{1+1}{1+2} C_2 = 0$$

Since  $C_2 = 0$  we have  $C_3 = C_4 = C_5 = C_6 = \dots = 0$ .

Thus one solution is:  $y_1 = 1 + 0 + 0 + \dots$

$$\Rightarrow \boxed{y_1 = 1}$$

\* Choose  $C_0 = 0$  and  $C_1 = 1$  we get:

$$C_2 = \frac{1}{2} \cdot 1 = \frac{1}{2}$$

$$k=1 \Rightarrow C_3 = \frac{2}{3} C_2 = \frac{1}{3}$$

$$k=2 \Rightarrow C_4 = \frac{1}{4}$$

$$\text{So } y_2 = x + \frac{1}{2} x^2 + \frac{1}{3} x^3 + \frac{1}{4} x^4 + \dots = \sum_{n=1}^{\infty} \frac{x^n}{n}$$

Hence we have two solutions  $y_1$  and  $y_2$  of the DE (1).