December 15,2010

Test♯1 Math202, sec 07

Net Time Allowed: 40 minutes

Name: **ID ♯:** **section:**

Exercise1: (05 pts)

Find the Cauchy-Euler equation whose solution is given by

$$y = c_1 x^{-1} + c_2 \cos(\sqrt{3}\ln x) + c_3 \sin(\sqrt{3}\ln x)$$

solution:

The roots are $m_1 = -1$, $m_2 = \sqrt{3}i$ and $m_3 = -\sqrt{3}i$ 01

So The auxi-eq is: $(m+1)(m^2+3) = 0 = m^3+m^2+3m+3$

or $m^3+m^2+3m+3 = m(m^2-3m+2)+4m^2+ \quad m+3 = 0$ 01

$= m(m-1)(m-2)+4m(m-1)+5m+3$ 01

Hence The Cauchy-Euler eq. is given by:

$$x^3y'''+4x^2y''+5xy'+3y=0$$ 02

Exercise2: (05 pts).

Using the method of undetermined Coefficients solve The DE: $y'' + 25y = 6\sin x$

solution: we have $m^2+25=0 \Rightarrow m = \pm 5i$

So $y_c = c_1\cos 5x + c_2 \sin 5x$, 01

• $(D^2+1)(D^2+25)y = 0 \Rightarrow (m^2+1)(m^2+25) = 0$ 01

The G.S is given by: $y = c_1\cos 5x + c_2\sin 5x + c_3\cos x + c_4 \sin x$ 01

So $y_p = A\cos x + B\sin x$ 01 (substituting $\Rightarrow A = 0$ and $B = \frac{1}{4}$)

Hence $y = c_1\cos 5x + c_2\sin 5x + \frac{1}{4}\sin x$ 01

Exercise3: (07 pts)

Consider the second order differential equation
(1)

$$y'' - 4y = 12.$$

Let $y_1(x) = e^{2x}$ be a solution of the associated homogenous equation of (1).
1)-Use the <u>method</u> of reduction of order to find a second solution $y_2(x)$.(**Do not use the Formula !**)
2)- Find the general solution of the equation (1).

solution:

Let $y = u(x)y_1(x) = u(x)e^{2x}$ 01 Then $y' = (2u+u')e^{2x}$ and $y'' = 2(u'+u'')e^{2x} + 2(2u+u')e^{2x}$ 01

$\Rightarrow y'' = (u''+4u'+4u)e^{2x}$ and $y''-4y=0 \Rightarrow$

$(u''+4u')e^{2x} = 0 \Rightarrow u''+4u'=0 \overset{w=u'}{\Longrightarrow} w'+4w=0$ 01

$w = c_1e^{-4x} \Rightarrow u = \int c_1e^{-4x}dx = c_2e^{-4x}$ (choose $c_2=1$) $\Rightarrow y_2 = e^{-2x}$ 01

Thus $y_c = c_1e^{2x} + c_2e^{-x}$ 01, clearly $y_p = -3$ is a parti- solution 01

Hence $y = y_c + y_p = c_1e^{2x} + c_2e^{-2x} - 3$ 01

Exercise4: (03 pts)

Solve $y''' - 5y'' + 3y' + 9y = 0$.

solution:

Auxi-eq: $m^3 - 5m^2 + 3m + 9 = 0$ 01

$\Rightarrow (m+1)(m-3)^2 = 0 \Rightarrow m_1 = -1,\ m_2 = 3$ (Repeated 2 times) 01

Hence The G.S is $y = c_1e^{-x} + c_2e^{3x} + c_3xe^{3x}$ 01