October 30,2010

QUIZ♯2 Math202

Net Time Allowed: 20 minutes

Name: **ID ♯:** **section:**

Exercise1: (0.6 pts)

Consider the differential equation: $y(y^2 \cos x+1)\,dx+(y^2 \sin x-x+y)\,dy=0.$ (1)

(a) Is the DE (1) exact? Justify your answer !

(b) Find an integrating factor which makes the DE (1) exact.

(c) Solve the DE (1).

solution:

a/ Let $M(x,y)=y(y^2\cos x+1)$ and $N(x,y)=y^2\sin x-x+y$ (01)

Since $\frac{\partial M}{\partial y}=3y^2\cos x+1$ and $\frac{\partial N}{\partial x}=y^2\cos x-1$, (DE) (1) is not exact

b/ $\frac{N_x-M_y}{M}=-\frac{2}{y}$ (0.5) which depend only of y. Hence an integrating factor is given by: $e^{-\int\frac{2}{y}dy}=e^{-\ln y^2}=\frac{1}{y^2}$. (01)

c/ Multiplying The (DE) (1) by the integrating factor, gives: (0.5)

$(y\cos x+\frac{1}{y})\,dx+(\sin x-\frac{x}{y^2}+\frac{1}{y})\,dy=0$, which is exact!

Let $f(x,y)$ be a function such that: $\frac{\partial f}{\partial x}=y\cos x+\frac{1}{y}$ (2) (0.5) and $\frac{\partial f}{\partial y}=\sin x-\frac{1}{y^2}+\frac{1}{y}$ (0.5) (3)

Integrate (2) % x: $f(x,y)=y\sin x+\frac{x}{y}+g(y)$ (4). Differentiate (4) % y gives:

$\frac{\partial f}{\partial y}=\sin x-\frac{x}{y^2}+g'(y)=\sin x-\frac{x}{y^2}+\frac{1}{y}$. Thus $g'(y)=\frac{1}{y}$. (01)

and So $g(y)=\ln|y|+C$, (0.5) Therefore $y\sin x+\frac{x}{y}+\ln|y|=C$ (01) is an implicit Solution of The (DE) (1).

Exercise2: (03.5 pts)

Solve the DE: $xy'+6y=3xy^{\frac{4}{3}}.$

Solution of Exercise 2 : (03.5 pts)

- This DE is of Bernoulli type with $n = \frac{4}{3}$. (0.5)
- Let $v = y^{1-n} = y^{-\frac{1}{3}} \Rightarrow y = v^{-3}$, (0.5) $\frac{dy}{dx} = -3v^{-4}\frac{dv}{dx}$. (0.5)

So $-3x\,v^{-4}\frac{dv}{dx} + 6v^{-3} = 3x\,v^{-4}$

(0.5)

or $\frac{dv}{dx} - \frac{2}{x}v = -1$

The integrating factor is $e^{-2\int\frac{dx}{x}} = x^{-2}$ (0.5)

Thus $\frac{d}{dx}\left[x^{-2}v\right] = -\frac{1}{x^2} \Rightarrow x^{-2}v = \frac{1}{x} + C$

(0.5)

$\Rightarrow v = x + Cx^2$

Hence $\boxed{y = \frac{1}{(x + Cx^2)^3}}$ (0.5)