MATH 202.6 (101)
Quiz 4 (Chaps. 4.6-4.7) **Duration: 20mn**

Name: **ID number:**

1.)(5pts)Find the general solution of the DE $2x^2y'' + 7xy' + 3y = 2x^3$.
2.)(5pts)Solve the DE $xy''' + y'' = 0$

Solution

1) $y = y_c + y_p$

To find y_c, we solve the homogeneous DE $2x^2y'' + 7xy' + 3y = 0$

The auxiliary equation is

$2m^2 + 5m + 3 = 0$

$\Delta = 1$, $m = \frac{-5-1}{4} = -3/2$

$m = \frac{-5+1}{4} = -1$

$y_c = c_1 x^{-1} + c_2 x^{-\frac{3}{2}}$

To find y_p we use variation of parameters.

Let $y_1 = x^{-1}$, $y_2 = x^{-3/2}$

$y_p = u_1 y_1 + u_2 y_2$

$W = \begin{vmatrix} x^{-1} & x^{-3/2} \\ -x^{-2} & -\frac{3}{2}x^{-5/2} \end{vmatrix} = -\frac{1}{2}x^{-7/2}$

$u_1' = -\frac{x^{-3/2}(x)}{-\frac{1}{2}x^{-7/2}} = 2x^3 \Rightarrow u_1 = \frac{x^4}{2}$

$u_2' = \frac{x^{-1}\cdot x}{-\frac{1}{2}x^{-7/2}} = -2x^{7/2} \Rightarrow u_2 = -\frac{4}{9}x^{9/2}$

$y_p = \frac{x^3}{2} - \frac{4}{9}x^3 = \frac{x^3}{18}$

$$y = c_1 x^{-1} + c_2 x^{-3/2} + \frac{x^3}{18}$$

2) Let $y = x^m$

$x\, m(m-1)(m-2)x^{m-3} + m(m-1)x^{m-2} = 0$

$x^{m-2}\left[m^3 - 3m^2 + 2m + m^2 - m\right] = 0$

$m^3 - 2m^2 + m = 0$

$m(m-1)^2 = 0$

$m = 0$, $m = 1$

$$y = c_1 + c_2 x + c_3 x \ln x$$