MATH 202.6 (101)

Quiz 1 (Chap. 1.1-2.3) Duration: 15mn

Name: ID number:

1.)(5pts)Solve the linear DE: $(1+t)\frac{dx}{dt}+x=(1+t)e^t$.

2.)(5pts)Solve the separable DE: $(x^2-4)\cos^2 y dy = dx$.

Solution

1) $(1+t)\frac{dx}{dt}+x=(1+t)e^t$

$\frac{dx}{dt}+\frac{1}{1+t}x=e^t$, $t\neq -1$

An integrating factor is

$e^{\int \frac{1}{1+t}dt} = e^{\ln|1+t|} = 1+t$, $t>-1$

We multiply the equation by $1+t$, and we find

$\frac{d}{dt}\left(x(1+t)\right) = (1+t)e^t$

and $x(1+t) = \int (1+t)e^t + C$

$x(1+t) = te^t + C$

$$\boxed{x = \frac{te^t+C}{1+t},\ t>-1}$$

2) $\cos^2 y\, dy = \frac{dx}{x^2-4}$, $x\neq \pm 2$

$\int \cos^2 y\, dy = \int \frac{dx}{x^2-4}$

$\int \frac{\cos 2y+1}{2} dy = \int \left(\frac{\frac{1}{4}}{x-2} - \frac{\frac{1}{4}}{x+2}\right) dx$

$\frac{1}{2}\left[\frac{\sin 2y}{2}+y\right] = \frac{1}{4}\ln|x-2| - \frac{1}{4}\ln|x+2| + C$

$\frac{\sin 2y}{2}+y = \frac{1}{2}\ln\left|\frac{x-2}{x+2}\right| + C$

$$\boxed{\frac{\sin 2y}{2}+y = \frac{1}{2}\ln\left(\frac{x-2}{x+2}\right)+C,\ x>2 \text{ or } x<-2}$$