**Quiz 4 (Chaps. 4.6-4.7)**      Duration: 20mn

Name:                          ID number:

1.)(5pts)Find the general solution of the DE $xy'' - 4y' = x^4$.

2.)(5pts)Solve the DE $x^3y''' + 4x^2y'' + 5xy' + 3y = 0$.

### Solution

**1)** $y = y_c + y_p$

First, we solve the homogeneous DE

$xy'' - 4y' = 0$

$\Rightarrow x^2y'' - 4xy' = 0$

The auxiliary equation is

$m^2 - 5m = 0$

$m = 0, \quad m = 5$

$y_c = c_1 + c_2 x^5$.

Now, we search for $y_p$.

let $y_1 = 1$ and $y_2 = x^5$.

$u_p = u_1 y_1 + u_2 y_2$

$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} 1 & x^5 \\ 0 & 5x^4 \end{vmatrix} = 5x^4$.

$u_1' = -\dfrac{x^5(x^3)}{5x^4} = -\dfrac{x^4}{5} \Rightarrow u_1 = -\dfrac{x^5}{25}$

$u_2' = \dfrac{1(x^3)}{5x^4} = \dfrac{1}{5x} \Rightarrow u_2 = \dfrac{1}{5}\ln x$.

$y_p = -\dfrac{x^5}{25} + \dfrac{x^5 \ln x}{5}$.

$$\Rightarrow \boxed{y = c_0 + c_2 x^5 - \dfrac{x^5}{25} + \dfrac{x^5 \ln x}{5}}$$

**2)** $y = x^m$

$x^3 m(m-1)(m-2)x^{m-3} + 4x^2 m(m-1)x^{m-2} +$

$5x m x^{m-1} + 3x^m = 0$

$x^m\left[m^3 - 3m^2 + 2m + 4m^2 - 4m + 5m + 3\right] = 0$

$m^3 + m^2 + 3m + 3 = 0$

$(m+1)(m^2+3) = 0$

$m = -1, \quad m = \pm i\sqrt{3}$

$$\boxed{y = c_1 x^{-1} + c_2 \cos(\sqrt{3}\ln x) + c_3 \sin(\sqrt{3}\ln x)}$$