MATH 202.1 (101)

Quiz 3 (Chaps. 4.1.3-4.5) Duration: 20mn

Name: ID number:

1.)(3pts)Knowing that $y_1 = e^x$ is a solution to the DE

$$y'' - 2y' + y = 0,$$

find a second solution y_2 linearly independent to y_1 by using only the method of reduction of order.

2.)(7pts)Solve the following DE by using only the annihilator approach

$$y'' + y' + y = \cos x.$$

Solution

1) $y_2 = y_1 \int \frac{e^{-\int P(x)}}{y_1^2} dx$.

$y'' + P(x) y' + Q(x) y = 0$

$\Rightarrow P(x) = -2$ and

$e^{-\int P(x)dx} = e^{2\int dx} = e^{2x}$

$y_2 = e^x \int \frac{e^{2x}}{e^{2x}} dx = x e^x$

$\underline{y_2 = x e^x}$.

2) $y = y_c + y_p$

First, we solve $y'' + y' + y = 0$

$m^2 + m + 1 = 0$

$m = \frac{-1 \pm i\sqrt{3}}{2}$

$y_c = e^{-\frac{x}{2}}\left(c_1 \cos \frac{\sqrt{3}}{2}x + c_2 \sin \frac{\sqrt{3}}{2}x\right)$

Now, we can write

$(D^2 + D + 1)\, y = \cos x$.

We know that $(D^2+1)(\cos x) = 0$

$(D^2 + D + 1)(D^2 + 1)\, y = 0$

$(m^2 + m + 1)(m^2 + 1) = 0$

$m = -\frac{1}{2} \pm i\frac{\sqrt{3}}{2}$

$m = \pm i$

$y = e^{-x/2}\left(c_1 \cos \frac{\sqrt{3}}{2}x + c_2 \sin \frac{\sqrt{3}}{2}x\right) + c_3 \cos x + c_4 \sin x$

$y_p = A \cos x + B \sin x$

$y_p'' + y_p' + y_p = \cos x \Rightarrow -A \sin x + B \cos x = \cos x$

$\Rightarrow A = 0,\ B = 1$

$\Rightarrow y_p = \sin x$.

$$\boxed{y = e^{-\frac{x}{2}}\left(c_1 \cos \frac{\sqrt{3}}{2}x + c_2 \sin \frac{\sqrt{3}}{2}x\right) + \sin x}$$