MATH 202.1 (101)

Quiz 1 (Chap. 1.1-2.3) Duration: 15mn

Name: ID number:

1.)(5pts)Solve the linear DE: $\sin x \frac{dy}{dx} - y\cos x = \sin^2 x,\ 0 < x < \frac{\pi}{2}$.

2.)(5pts)Solve the separable DE: $y^2 \frac{dy}{dx} = x^2 e^{x-y^3}$.

Solution

1°) $\sin x \frac{dy}{dx} - y\cos x = \sin^2 x$

$\frac{dy}{dx} - \frac{\cos x}{\sin x} y = \sin x, \quad 0 < x < \frac{\pi}{2}$

An integrating factor is

$e^{-\int \frac{\cos x}{\sin x} dx} = e^{-\ln(\sin x)} = \frac{1}{\sin x}, \quad 0 < x < \frac{\pi}{2}.$

We multiply the equation by $\frac{1}{\sin x}$

$\frac{d}{dx}\left(\frac{y}{\sin x}\right) = 1$

$\frac{y}{\sin x} = \int 1 dx + C$

$\frac{y}{\sin x} = x + C$

$$\boxed{y = (x+C)\sin x, \quad 0 < x < \frac{\pi}{2}}$$

2) $y^2 \frac{dy}{dx} = x^2 e^{x-y^3}$

$y^2 e^{y^3} dy = x^2 e^x$

$\int y^2 e^{y^3} dy = \int x^2 e^x$

$\frac{e^{y^3}}{3} = (x^2 - 2x + 2) e^x + C$

$$\boxed{e^{y^3} = 3(x^2 - 2x + 2) e^x + C}$$