

King Fahd University of Petroleum and Minerals
Department of Mathematics & Statistics
Fall 2011 (101)-Math 201 - (19)-Class Test (4)

Time: 2 hours

Name: _____ ID #: _____ Serial #: _____

No Calculator is Allowed in this Test
Show All Necessary Work

1	/10
2	/10
3	/10
4	/5
5	/15
6	/10
7	/10
8	/5
9	/10
10	/15
Total	/100

(1)(a) Find $\lim_{(x,y) \rightarrow (0,0)} \frac{x^3 y}{x^6 + y^2}$ if it exists.

(b) Find $D_u f(2, 0)$ where $f(x, y) = xe^{xy} + y$ and u is the unit vector in the direction of $\theta = \frac{2\pi}{3}$.

(2) Compute $\frac{\partial^2 z}{\partial \theta^2}$ for $z = f(x, y)$ if $x = r \cos \theta$, and $y = r \sin \theta$.

(3) Find the absolute maximum and minimum of $f(x, y) = x^2 + xy + y^2$ on the disk $\{(x, y) | x^2 + y^2 \leq 9\}$.

(4) **DO EITHER (a) OR (b)**

(a) The length and width of a rectangle are measured as 30 cm and 24 cm, respectively, with an error in measurement of at most 0.1 cm in each. Use differentials to estimate the maximum error in calculated area of the rectangle.

(b) Find the linear approximation of the function $f(x, y) = \sqrt{20 - x^2 - 7y^2}$ at $(2, 1)$ and use it to approximate $f(1.95, 1.08)$.

- (5) (a) Let $z = x - \tan^{-1}(yz)$. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.
- (b) Find the tangent plane and normal line to $x^2 + y^2 + z^2 = 30$ at the point $(1, -2, 5)$.

(6) DO EITHER (a) OR (b)

(a) Find the volume of the solid in the first octant bounded by the cylinder $z = 16 - x^2$ and the plane $y = 5$.

(b) Find the volume inside the sphere $x^2 + y^2 + z^2 = 25$ and outside the cylinder $x^2 + y^2 = 9$.

(7) Given that $\int_0^{\pi/2} \frac{dx}{1+\sin^2 x} = \frac{\pi}{2\sqrt{2}}$.

(a) Evaluate the double integral $\int_0^{\pi/2} \int_0^{\pi/2} \frac{1}{(1+\sin^2 x)(1+\sin^2 y)} dx dy$.

(b) Evaluate the triple integral $\int_0^{\pi/2} \int_0^{\pi/2} \int_{1/(1+\sin^2 x)}^{1/(1+\sin^2 y)} dz dx dy$.

(8) Evaluate the double integral $\int_0^8 \int_{\sqrt[3]{y}}^2 e^{x^4} dx dy$.

(9) If $E = \{(x, y, z) : x^2 + y^2 \leq z \leq 4x\}$, express the volume of E (BUT DO NOT EVALUATE) as a triple integral in (a) rectangular coordinates and (b) cylindrical coordinates.

(10) (a) Find an equivalent integral of the form $\int_E f(x, y, z) dx dz dy$ for $\int_0^1 \int_0^{(1-x)/2} \int_0^{1-x-2y} z dz dy dx$.

(b) Change to spherical coordinates and evaluate:
 $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{1}{1+x^2+y^2+z^2} dz dy dx$.

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