1. An equation for the tangent line to the polar curve

$$r = 3\sin 3\theta$$

at $\theta = \dfrac{\pi}{6}$ is

(a) $y = 6 - \sqrt{3}\,x$

(b) $y = 6 - \dfrac{\pi}{6}\,x$

(c) $y = \dfrac{\pi}{6} - \sqrt{3}\,x$

(d) $y = 2\sqrt{3}\,x + 6$

(e) $y = 6 - 3x$

2. The length of the curve represented by

$$x = 2 + 3t, y = \cosh 3t, \quad 0 \le t \le 1$$

is equal to

(a) $\sinh 3$

(b) $\cosh 3$

(c) $(1 + \cosh 3)$

(d) $(1 + \sinh 3)$

(e) $\cosh 3 + \sinh 3$

3. The area of the region that lies inside both of the curves $r = 2\sin\theta$ and $r = \sin\theta + \cos\theta$ is equal to

(a) $\frac{1}{2}(\pi - 1)$

(b) $\frac{\pi}{2}$

(c) $2\pi + 1$

(d) $\pi + \frac{1}{2}$

(e) $\pi - \frac{1}{2}$

4. Let $\vec{u} = \langle 1, -1, 0\rangle$ and $\vec{v} = \langle 0, 2, -1\rangle$. If θ is the angle between $\vec{u}$ and $\vec{v}$, then $\tan\theta$ is equal to

(a) $-\frac{\sqrt{6}}{2}$

(b) $-\sqrt{6}$

(c) $\frac{\sqrt{3}}{\sqrt{2}}$

(d) $\frac{\sqrt{3}}{\sqrt{5}}$

(e) $-\frac{2}{\sqrt{6}}$

5. The equation $3x^2 - 2y^2 + 3z^2 + 30x - 8y - 24z + 131 = 0$ represents

(a) A hyperboloid of two sheets

(b) A cone

(c) A hyperboloid of one sheet

(d) A hyperbolic paraboloid

(e) An elliptic paraboloid

6. The distance from the plane $(x-1)+(y-2)+2(z-5)=0$ to the plane $x+y+2z=4$ is equal to

(a) $9/\sqrt{6}$

(b) $4/\sqrt{6}$

(c) $2/\sqrt{6}$

(d) $13/\sqrt{6}$

(e) 9

7. Given that $x = 2s^2 + 3t,\ y = 3s - 2t^2,\ z = f(x, y),\ f_x(-1, 1) = 7,$ $f_y(-1, 1) = -3$. The value of $\dfrac{\partial z}{\partial s} + \dfrac{\partial z}{\partial t}$ at $s = 1,\ t = -1$ is

(a) 28

(b) 27

(c) 4

(d) 5

(e) -1

8. The absolute minimum of $f(x, y) = 6 + 3xy - 2x - 4y$ on the region D bounded by the parabola $y = x^2$ and the line $y = 4$ is equal to

(a) -30

(b) 0

(c) -34

(d) -36

(e) 30

9. The equation of the tangent plane to the surface $z + 5 = xe^y \cos z$ at the point $(5, 0, 0)$ is

(a) $x + 5y - z = 5$

(b) $x + 5y + z = 5$

(c) $x + y - 5z = 5$

(d) $5x + y - z = 25$

(e) $x + y - z = 5$

10. If $f(x, y) = y^3 - 6xy + 8x^2$, then a local minimum of f occurs at

(a) $\left(\frac{9}{32}, \frac{3}{4}\right)$

(b) $(0, 0)$

(c) $(9, 3)$

(d) $(32, 4)$

(e) $\left(\frac{9}{2}, \frac{4}{3}\right)$

11. The maximum rate of change of $f(x,y) = xye^{-xy^2}$ at the point $(1,1)$ is

(a) e^{-1}

(b) e^{-4}

(c) e

(d) $\frac{e^{-1}}{2}$

(e) 1

12. The maximum value of $f(x,y) = x + 2y$ subject to the constraint $x^2 + y^2 = 1$ is equal to

(a) $\sqrt{5}$

(b) $\frac{1}{\sqrt{5}}$

(c) $\frac{2}{\sqrt{5}}$

(d) $2\sqrt{5}$

(e) $\frac{3}{\sqrt{5}}$

13. The rate of change of $f(x, y) = \sqrt{6x - 5y}$ at $(5, 1)$ in the direction of a vector at an angle of $-\pi/6$ from the positive x-axis is

(a) $\frac{1}{4} + \frac{3\sqrt{3}}{10}$

(b) $\frac{1}{6} + \frac{\sqrt{3}}{10}$

(c) $\frac{3\sqrt{3}}{10}$

(d) $5 + \frac{\sqrt{3}}{10}$

(e) $1 + \frac{\sqrt{5}}{10}$

14. If the point $(4, 4\sqrt{3}, -7)$ is given in rectangular coordinates, then its cylindrical coordinates are given by

(a) $\left(-8, \frac{4\pi}{3}, -7\right)$

(b) $\left(8, \frac{2\pi}{3}, -7\right)$

(c) $\left(8, \frac{2\pi}{3}, 7\right)$

(d) $\left(-8, \frac{\pi}{3}, -7\right)$

(e) $\left(8, \frac{5\pi}{3}, 7\right)$

15. Using four equal squares and their midpoints, the best estimate for the volume of the solid that lies above the square

$$R = \{(x, y) |\ 0 \le x \le 4, \quad 0 \le y \le 4\}$$

and below the elliptic paraboloid $f(x, y) = 68 - 2x^2 - 2y^2$ is

(a) 768

(b) 836

(c) 778

(d) 192

(e) 762

16. The volume under the surface $z = x^5 + y^5$ and above the region bounded by $y = x^2$ and $x = y^2$ is equal to

(a) $\frac{3}{52}$

(b) $\frac{1}{37}$

(c) $\frac{1}{32}$

(d) $\frac{3}{32}$

(e) $\frac{1}{18}$

17. The double integral

$$\int_0^{\sqrt{2}} \int_y^{\sqrt{4-y^2}} \frac{1}{1+x^2+y^2} dx\, dy$$

is equal to

(a) $\frac{\pi}{8}\ln 5$

(b) $\frac{\pi}{4}\ln 3$

(c) $\frac{\pi}{3}\ln 5$

(d) $\pi \ln 5$

(e) $\sqrt{2}\,\pi$

18. Evaluate the iterated integral

$$\int_0^1 \int_{x^2}^1 \sqrt{y}\sin(x\sqrt{y})dy\, dx.$$

(a) $1 - \sin 1$

(b) $\sin 1$

(c) $1 - \sqrt{\sin 1}$

(d) $1 - \cos 1$

(e) $\cos 1$

19. The volume of the solid Q lying inside both the sphere $x^2 + y^2 + z^2 = 2z$ and the cone $z^2 = x^2 + y^2$ is given in spherical coordinates by

(a) $\int_0^{2\pi} \int_0^{\pi/4} \int_0^{2\cos\phi} \rho^2 \sin\phi d\rho d\phi d\theta$

(b) $\int_0^{2\pi} \int_0^{\pi/2} \int_0^{1/2\cos\theta} \rho^2 \sin\phi d\rho d\phi d\theta$

(c) $\int_0^{\pi/2} \int_0^{\pi/4} \int_0^{2\pi} \rho^2 \sin\phi d\rho d\phi d\theta$

(d) $\int_0^{2\pi} \int_0^{\pi/4} \int_0^{2\cos\phi} \rho \sin^2\phi d\rho d\phi d\theta$

(e) $\int_0^{\pi} \int_0^{2\pi} \int_0^{2\sin\phi} \rho^2 \sin\phi d\rho d\phi d\theta$

20. The volume of the solid above the xy-plane inside both the sphere $x^2 + y^2 + z^2 = 16$ and the cylinder $x^2 + y^2 - 4x = 0$ is equal to

(a) $\frac{64}{9}(3\pi - 4)$

(b) $\frac{64\pi}{3}$

(c) $\frac{\pi}{9} + 4$

(d) $\frac{3\pi}{4} + \frac{64}{9}$

(e) $\frac{64}{9}\pi$

1. The absolute minimum of $f(x, y) = 6 + 3xy - 2x - 4y$ on the region D bounded by the parabola $y = x^2$ and the line $y = 4$ is equal to

(a) 0

(b) -30

(c) -34

(d) -36

(e) 30

2. The maximum rate of change of $f(x, y) = xye^{-xy^2}$ at the point $(1, 1)$ is

(a) e^{-4}

(b) $\dfrac{e^{-1}}{2}$

(c) e^{-1}

(d) e

(e) 1

3. The rate of change of $f(x, y) = \sqrt{6x - 5y}$ at $(5, 1)$ in the direction of a vector at an angle of $-\pi/6$ from the positive x-axis is

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(d) $5 + \frac{\sqrt{3}}{10}$

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5. The maximum value of $f(x, y) = x + 2y$ subject to the constraint $x^2 + y^2 = 1$ is equal to

(a) $\sqrt{5}$

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(c) $\dfrac{1}{\sqrt{5}}$

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6. The volume of the solid above the xy-plane inside both the sphere $x^2 + y^2 + z^2 = 16$ and the cylinder $x^2 + y^2 - 4x = 0$ is equal to

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7. Let $\vec{u} = \langle 1, -1, 0\rangle$ and $\vec{v} = \langle 0, 2, -1\rangle$. If θ is the angle between $\vec{u}$ and $\vec{v}$, then $\tan\theta$ is equal to

(a) $\dfrac{\sqrt{3}}{\sqrt{2}}$

(b) $\dfrac{\sqrt{3}}{\sqrt{5}}$

(c) $-\dfrac{2}{\sqrt{6}}$

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8. The distance from the plane $(x-1)+(y-2)+2(z-5)=0$ to the plane $x+y+2z=4$ is equal to

(a) $13/\sqrt{6}$

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10. The double integral

$$\int_0^{\sqrt{2}} \int_y^{\sqrt{4-y^2}} \frac{1}{1+x^2+y^2} dx\, dy$$

is equal to

(a) $\sqrt{2}\,\pi$

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(c) $\frac{\pi}{3} \ln 5$

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(a) 28

(b) 5

(c) 4

(d) -1

(e) 27

14. The area of the region that lies inside both of the curves $r = 2\sin\theta$ and $r = \sin\theta + \cos\theta$ is equal to

(a) $\pi + \dfrac{1}{2}$

(b) $\dfrac{\pi}{2}$

(c) $\dfrac{1}{2}(\pi - 1)$

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(e) $2\pi + 1$

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(d) $x + 5y - z = 5$

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19. The length of the curve represented by

$$x = 2 + 3t, y = \cosh 3t, \quad 0 \leq t \leq 1$$

is equal to

(a) $(1 + \sinh 3)$

(b) $\cosh 3 + \sinh 3$

(c) $(1 + \cosh 3)$

(d) $\cosh 3$

(e) $\sinh 3$

20. If $f(x, y) = y^3 - 6xy + 8x^2$, then a local minimum of f occurs at

(a) $(9, 3)$

(b) $(32, 4)$

(c) $\left(\frac{9}{2}, \frac{4}{3}\right)$

(d) $\left(\frac{9}{32}, \frac{3}{4}\right)$

(e) $(0, 0)$

Name ....

ID .... Sec ....

1	a	b	c	d	e	f
2	a	b	c	d	e	f
3	a	b	c	d	e	f
4	a	b	c	d	e	f
5	a	b	c	d	e	f
6	a	b	c	d	e	f
7	a	b	c	d	e	f
8	a	b	c	d	e	f
9	a	b	c	d	e	f
10	a	b	c	d	e	f
11	a	b	c	d	e	f
12	a	b	c	d	e	f
13	a	b	c	d	e	f
14	a	b	c	d	e	f
15	a	b	c	d	e	f
16	a	b	c	d	e	f
17	a	b	c	d	e	f
18	a	b	c	d	e	f
19	a	b	c	d	e	f
20	a	b	c	d	e	f
21	a	b	c	d	e	f
22	a	b	c	d	e	f
23	a	b	c	d	e	f
24	a	b	c	d	e	f
25	a	b	c	d	e	f
26	a	b	c	d	e	f
27	a	b	c	d	e	f
28	a	b	c	d	e	f
29	a	b	c	d	e	f
30	a	b	c	d	e	f
31	a	b	c	d	e	f
32	a	b	c	d	e	f
33	a	b	c	d	e	f
34	a	b	c	d	e	f
35	a	b	c	d	e	f

36	a	b	c	d	e	f
37	a	b	c	d	e	f
38	a	b	c	d	e	f
39	a	b	c	d	e	f
40	a	b	c	d	e	f
41	a	b	c	d	e	f
42	a	b	c	d	e	f
43	a	b	c	d	e	f
44	a	b	c	d	e	f
45	a	b	c	d	e	f
46	a	b	c	d	e	f
47	a	b	c	d	e	f
48	a	b	c	d	e	f
49	a	b	c	d	e	f
50	a	b	c	d	e	f
51	a	b	c	d	e	f
52	a	b	c	d	e	f
53	a	b	c	d	e	f
54	a	b	c	d	e	f
55	a	b	c	d	e	f
56	a	b	c	d	e	f
57	a	b	c	d	e	f
58	a	b	c	d	e	f
59	a	b	c	d	e	f
60	a	b	c	d	e	f
61	a	b	c	d	e	f
62	a	b	c	d	e	f
63	a	b	c	d	e	f
64	a	b	c	d	e	f
65	a	b	c	d	e	f
66	a	b	c	d	e	f
67	a	b	c	d	e	f
68	a	b	c	d	e	f
69	a	b	c	d	e	f
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1. The volume of the solid Q lying inside both the sphere $x^2 + y^2 + z^2 = 2z$ and the cone $z^2 = x^2 + y^2$ is given in spherical coordinates by

(a) $\int_0^{\pi} \int_0^{2\pi} \int_0^{2\sin\phi} \rho^2 \sin\phi d\rho d\phi d\theta$

(b) $\int_0^{\pi/2} \int_0^{\pi/4} \int_0^{2\pi} \rho^2 \sin\phi d\rho d\phi d\theta$

(c) $\int_0^{2\pi} \int_0^{\pi/2} \int_0^{1/2\cos\theta} \rho^2 \sin\phi d\rho d\phi d\theta$

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(e) $\int_0^{2\pi} \int_0^{\pi/4} \int_0^{2\cos\phi} \rho \sin^2\phi d\rho d\phi d\theta$

2. If the point $(4, 4\sqrt{3}, -7)$ is given in rectangular coordinates, then its cylindrical coordinates are given by

(a) $\left(8, \frac{5\pi}{3}, 7\right)$

(b) $\left(8, \frac{2\pi}{3}, -7\right)$

(c) $\left(-8, \frac{\pi}{3}, -7\right)$

(d) $\left(8, \frac{2\pi}{3}, 7\right)$

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3. The volume under the surface $z = x^5 + y^5$ and above the region bounded by $y = x^2$ and $x = y^2$ is equal to

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(a) $(9, 3)$

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and below the elliptic paraboloid $f(x, y) = 68 - 2x^2 - 2y^2$ is

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6. The maximum rate of change of $f(x, y) = xye^{-xy^2}$ at the point $(1, 1)$ is

(a) e

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(c) 1

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(c) $\dfrac{1}{6} + \dfrac{\sqrt{3}}{10}$

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8. Let $\vec{u} = \langle 1, -1, 0 \rangle$ and $\vec{v} = \langle 0, 2, -1 \rangle$. If θ is the angle between $\vec{u}$ and $\vec{v}$, then $\tan\theta$ is equal to

(a) $-\sqrt{6}$

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(a) -30

(b) -34

(c) 0

(d) 30

(e) -36

10. The distance from the plane $(x-1)+(y-2)+2(z-5) = 0$ to the plane $x + y + 2z = 4$ is equal to

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(c) An elliptic paraboloid

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17. Given that $x = 2s^2 + 3t,\ y = 3s - 2t^2,\ z = f(x, y),\ f_x(-1, 1) = 7,$ $f_y(-1, 1) = -3$. The value of $\dfrac{\partial z}{\partial s} + \dfrac{\partial z}{\partial t}$ at $s = 1,\ t = -1$ is

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(b) 5

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(e) -1

18. The maximum value of $f(x, y) = x + 2y$ subject to the constraint $x^2 + y^2 = 1$ is equal to

(a) $2\sqrt{5}$

(b) $\sqrt{5}$

(c) $\dfrac{3}{\sqrt{5}}$

(d) $\dfrac{1}{\sqrt{5}}$

(e) $\dfrac{2}{\sqrt{5}}$

19. Evaluate the iterated integral

$$\int_0^1 \int_{x^2}^1 \sqrt{y}\sin(x\sqrt{y})dy\ dx.$$

(a) $1 - \cos 1$

(b) $\sin 1$

(c) $1 - \sqrt{\sin 1}$

(d) $1 - \sin 1$

(e) $\cos 1$

20. The equation of the tangent plane to the surface $z + 5 = xe^y \cos z$ at the point $(5, 0, 0)$ is

(a) $x + 5y + z = 5$

(b) $x + y - z = 5$

(c) $x + 5y - z = 5$

(d) $5x + y - z = 25$

(e) $x + y - 5z = 5$

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1	a	b	c	d	e	f	36	a	b	c	d	e	f
2	a	b	c	d	e	f	37	a	b	c	d	e	f
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15	a	b	c	d	e	f	50	a	b	c	d	e	f
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17	a	b	c	d	e	f	52	a	b	c	d	e	f
18	a	b	c	d	e	f	53	a	b	c	d	e	f
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25	a	b	c	d	e	f	60	a	b	c	d	e	f
26	a	b	c	d	e	f	61	a	b	c	d	e	f
27	a	b	c	d	e	f	62	a	b	c	d	e	f
28	a	b	c	d	e	f	63	a	b	c	d	e	f
29	a	b	c	d	e	f	64	a	b	c	d	e	f
30	a	b	c	d	e	f	65	a	b	c	d	e	f
31	a	b	c	d	e	f	66	a	b	c	d	e	f
32	a	b	c	d	e	f	67	a	b	c	d	e	f
33	a	b	c	d	e	f	68	a	b	c	d	e	f
34	a	b	c	d	e	f	69	a	b	c	d	e	f
35	a	b	c	d	e	f	70	a	b	c	d	e	f

1. Evaluate the iterated integral

$$\int_0^1 \int_{x^2}^1 \sqrt{y}\sin(x\sqrt{y})dy\ dx.$$

(a) $1 - \cos 1$

(b) $\cos 1$

(c) $\sin 1$

(d) $1 - \sqrt{\sin 1}$

(e) $1 - \sin 1$

2. The area of the region that lies inside both of the curves $r = 2\sin\theta$ and $r = \sin\theta + \cos\theta$ is equal to

(a) $\frac{\pi}{2}$

(b) $\pi + \frac{1}{2}$

(c) $\frac{1}{2}(\pi - 1)$

(d) $\pi - \frac{1}{2}$

(e) $2\pi + 1$

3. The volume under the surface $z = x^5 + y^5$ and above the region bounded by $y = x^2$ and $x = y^2$ is equal to

(a) $\frac{1}{32}$

(b) $\frac{1}{37}$

(c) $\frac{3}{52}$

(d) $\frac{1}{18}$

(e) $\frac{3}{32}$

4. If $f(x,y) = y^3 - 6xy + 8x^2$, then a local minimum of f occurs at

(a) $\left(\frac{9}{2}, \frac{4}{3}\right)$

(b) $(32, 4)$

(c) $(9, 3)$

(d) $(0, 0)$

(e) $\left(\frac{9}{32}, \frac{3}{4}\right)$

5. Let $\vec{u} = \langle 1, -1, 0\rangle$ and $\vec{v} = \langle 0, 2, -1\rangle$. If θ is the angle between $\vec{u}$ and $\vec{v}$, then $\tan\theta$ is equal to

(a) $\dfrac{\sqrt{3}}{\sqrt{2}}$

(b) $-\dfrac{2}{\sqrt{6}}$

(c) $\dfrac{\sqrt{3}}{\sqrt{5}}$

(d) $-\dfrac{\sqrt{6}}{2}$

(e) $-\sqrt{6}$

6. An equation for the tangent line to the polar curve

$$r = 3\sin 3\theta$$

at $\theta = \dfrac{\pi}{6}$ is

(a) $y = 6 - \dfrac{\pi}{6}x$

(b) $y = 6 - 3x$

(c) $y = \dfrac{\pi}{6} - \sqrt{3}\,x$

(d) $y = 6 - \sqrt{3}\,x$

(e) $y = 2\sqrt{3}\,x + 6$

7. The equation $3x^2 - 2y^2 + 3z^2 + 30x - 8y - 24z + 131 = 0$ represents

(a) A hyperboloid of two sheets

(b) A hyperboloid of one sheet

(c) A cone

(d) An elliptic paraboloid

(e) A hyperbolic paraboloid

8. The length of the curve represented by

$$x = 2 + 3t, y = \cosh 3t, \quad 0 \le t \le 1$$

is equal to

(a) $\sinh 3$

(b) $(1 + \cosh 3)$

(c) $\cosh 3 + \sinh 3$

(d) $(1 + \sinh 3)$

(e) $\cosh 3$

9. The rate of change of $f(x, y) = \sqrt{6x - 5y}$ at $(5, 1)$ in the direction of a vector at an angle of $-\pi/6$ from the positive x-axis is

(a) $5 + \dfrac{\sqrt{3}}{10}$

(b) $\dfrac{1}{6} + \dfrac{\sqrt{3}}{10}$

(c) $\dfrac{1}{4} + \dfrac{3\sqrt{3}}{10}$

(d) $\dfrac{3\sqrt{3}}{10}$

(e) $1 + \dfrac{\sqrt{5}}{10}$

10. The maximum value of $f(x, y) = x + 2y$ subject to the constraint $x^2 + y^2 = 1$ is equal to

(a) $\dfrac{3}{\sqrt{5}}$

(b) $2\sqrt{5}$

(c) $\dfrac{1}{\sqrt{5}}$

(d) $\sqrt{5}$

(e) $\dfrac{2}{\sqrt{5}}$

11. The double integral

$$\int_0^{\sqrt{2}} \int_y^{\sqrt{4-y^2}} \frac{1}{1+x^2+y^2} dx\, dy$$

is equal to

(a) $\frac{\pi}{3} \ln 5$

(b) $\frac{\pi}{8} \ln 5$

(c) $\pi \ln 5$

(d) $\frac{\pi}{4} \ln 3$

(e) $\sqrt{2}\, \pi$

12. The volume of the solid above the xy-plane inside both the sphere $x^2 + y^2 + z^2 = 16$ and the cylinder $x^2 + y^2 - 4x = 0$ is equal to

(a) $\frac{64}{9}(3\pi - 4)$

(b) $\frac{\pi}{9} + 4$

(c) $\frac{64\pi}{3}$

(d) $\frac{64}{9}\pi$

(e) $\frac{3\pi}{4} + \frac{64}{9}$

13. Using four equal squares and their midpoints, the best estimate for the volume of the solid that lies above the square

$$R = \{(x, y)|\ 0 \leq x \leq 4, \quad 0 \leq y \leq 4\}$$

and below the elliptic paraboloid $f(x, y) = 68 - 2x^2 - 2y^2$ is

(a) 778

(b) 768

(c) 192

(d) 836

(e) 762

14. The maximum rate of change of $f(x, y) = xye^{-xy^2}$ at the point $(1, 1)$ is

(a) $\dfrac{e^{-1}}{2}$

(b) e^{-4}

(c) 1

(d) e

(e) e^{-1}

15. The distance from the plane $(x-1)+(y-2)+2(z-5)=0$ to the plane $x+y+2z=4$ is equal to

(a) $13/\sqrt{6}$

(b) $2/\sqrt{6}$

(c) $4/\sqrt{6}$

(d) 9

(e) $9/\sqrt{6}$

16. Given that $x=2s^2+3t,\ y=3s-2t^2,\ z=f(x,y),\ f_x(-1,1)=7,$ $f_y(-1,1)=-3$. The value of $\dfrac{\partial z}{\partial s}+\dfrac{\partial z}{\partial t}$ at $s=1,\ t=-1$ is

(a) 4

(b) 27

(c) -1

(d) 28

(e) 5

17. The absolute minimum of $f(x,y) = 6 + 3xy - 2x - 4y$ on the region D bounded by the parabola $y = x^2$ and the line $y = 4$ is equal to

(a) -34

(b) -30

(c) 30

(d) -36

(e) 0

18. The equation of the tangent plane to the surface $z + 5 = xe^y \cos z$ at the point $(5, 0, 0)$ is

(a) $5x + y - z = 25$

(b) $x + y - 5z = 5$

(c) $x + y - z = 5$

(d) $x + 5y - z = 5$

(e) $x + 5y + z = 5$

19. If the point $(4, 4\sqrt{3}, -7)$ is given in rectangular coordinates, then its cylindrical coordinates are given by

(a) $\left(-8, \frac{4\pi}{3}, -7\right)$

(b) $\left(8, \frac{2\pi}{3}, 7\right)$

(c) $\left(-8, \frac{\pi}{3}, -7\right)$

(d) $\left(8, \frac{2\pi}{3}, -7\right)$

(e) $\left(8, \frac{5\pi}{3}, 7\right)$

20. The volume of the solid Q lying inside both the sphere $x^2 + y^2 + z^2 = 2z$ and the cone $z^2 = x^2 + y^2$ is given in spherical coordinates by

(a) $\int_0^{\pi/2} \int_0^{\pi/4} \int_0^{2\pi} \rho^2 \sin\phi d\rho d\phi d\theta$

(b) $\int_0^{\pi} \int_0^{2\pi} \int_0^{2\sin\phi} \rho^2 \sin\phi d\rho d\phi d\theta$

(c) $\int_0^{2\pi} \int_0^{\pi/2} \int_0^{1/2\cos\theta} \rho^2 \sin\phi d\rho d\phi d\theta$

(d) $\int_0^{2\pi} \int_0^{\pi/4} \int_0^{2\cos\phi} \rho^2 \sin\phi d\rho d\phi d\theta$

(e) $\int_0^{2\pi} \int_0^{\pi/4} \int_0^{2\cos\phi} \rho \sin^2\phi d\rho d\phi d\theta$

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64	a	b	c	d	e	f
65	a	b	c	d	e	f
66	a	b	c	d	e	f
67	a	b	c	d	e	f
68	a	b	c	d	e	f
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1. The rate of change of $f(x, y) = \sqrt{6x - 5y}$ at $(5, 1)$ in the direction of a vector at an angle of $-\pi/6$ from the positive x-axis is

(a) $\frac{1}{6} + \frac{\sqrt{3}}{10}$

(b) $\frac{3\sqrt{3}}{10}$

(c) $5 + \frac{\sqrt{3}}{10}$

(d) $\frac{1}{4} + \frac{3\sqrt{3}}{10}$

(e) $1 + \frac{\sqrt{5}}{10}$

2. The distance from the plane $(x - 1) + (y - 2) + 2(z - 5) = 0$ to the plane $x + y + 2z = 4$ is equal to

(a) $13/\sqrt{6}$

(b) $4/\sqrt{6}$

(c) 9

(d) $9/\sqrt{6}$

(e) $2/\sqrt{6}$

3. The area of the region that lies inside both of the curves $r = 2\sin\theta$ and $r = \sin\theta + \cos\theta$ is equal to

(a) $\frac{\pi}{2}$

(b) $\frac{1}{2}(\pi - 1)$

(c) $2\pi + 1$

(d) $\pi - \frac{1}{2}$

(e) $\pi + \frac{1}{2}$

4. The maximum rate of change of $f(x, y) = xye^{-xy^2}$ at the point $(1, 1)$ is

(a) e^{-4}

(b) $\frac{e^{-1}}{2}$

(c) 1

(d) e^{-1}

(e) e

5. The maximum value of $f(x, y) = x + 2y$ subject to the constraint $x^2 + y^2 = 1$ is equal to

(a) $\sqrt{5}$

(b) $2\sqrt{5}$

(c) $\dfrac{1}{\sqrt{5}}$

(d) $\dfrac{2}{\sqrt{5}}$

(e) $\dfrac{3}{\sqrt{5}}$

6. The equation $3x^2 - 2y^2 + 3z^2 + 30x - 8y - 24z + 131 = 0$ represents

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7. An equation for the tangent line to the polar curve

$$r = 3\sin 3\theta$$

at $\theta = \dfrac{\pi}{6}$ is

(a) $y = \dfrac{\pi}{6} - \sqrt{3}\,x$

(b) $y = 6 - 3x$

(c) $y = 6 - \dfrac{\pi}{6}\,x$

(d) $y = 2\sqrt{3}\,x + 6$

(e) $y = 6 - \sqrt{3}\,x$

8. The volume under the surface $z = x^5 + y^5$ and above the region bounded by $y = x^2$ and $x = y^2$ is equal to

(a) $\dfrac{3}{52}$

(b) $\dfrac{1}{18}$

(c) $\dfrac{3}{32}$

(d) $\dfrac{1}{37}$

(e) $\dfrac{1}{32}$

9. The absolute minimum of $f(x, y) = 6 + 3xy - 2x - 4y$ on the region D bounded by the parabola $y = x^2$ and the line $y = 4$ is equal to

(a) -36

(b) -30

(c) 30

(d) -34

(e) 0

10. The equation of the tangent plane to the surface $z + 5 = xe^y \cos z$ at the point $(5, 0, 0)$ is

(a) $x + 5y + z = 5$

(b) $5x + y - z = 25$

(c) $x + 5y - z = 5$

(d) $x + y - 5z = 5$

(e) $x + y - z = 5$

11. The volume of the solid above the xy-plane inside both the sphere $x^2 + y^2 + z^2 = 16$ and the cylinder $x^2 + y^2 - 4x = 0$ is equal to

(a) $\frac{64}{9}(3\pi - 4)$

(b) $\frac{64}{9}\pi$

(c) $\frac{3\pi}{4} + \frac{64}{9}$

(d) $\frac{64\pi}{3}$

(e) $\frac{\pi}{9} + 4$

12. The volume of the solid Q lying inside both the sphere $x^2 + y^2 + z^2 = 2z$ and the cone $z^2 = x^2 + y^2$ is given in spherical coordinates by

(a) $\int_0^{2\pi} \int_0^{\pi/4} \int_0^{2\cos\phi} \rho^2 \sin\phi d\rho d\phi d\theta$

(b) $\int_0^{\pi} \int_0^{2\pi} \int_0^{2\sin\phi} \rho^2 \sin\phi d\rho d\phi d\theta$

(c) $\int_0^{2\pi} \int_0^{\pi/2} \int_0^{1/2\cos\theta} \rho^2 \sin\phi d\rho d\phi d\theta$

(d) $\int_0^{\pi/2} \int_0^{\pi/4} \int_0^{2\pi} \rho^2 \sin\phi d\rho d\phi d\theta$

(e) $\int_0^{2\pi} \int_0^{\pi/4} \int_0^{2\cos\phi} \rho \sin^2\phi d\rho d\phi d\theta$

13. The double integral

$$\int_0^{\sqrt{2}} \int_y^{\sqrt{4-y^2}} \frac{1}{1+x^2+y^2} dx\, dy$$

is equal to

(a) $\frac{\pi}{4} \ln 3$

(b) $\sqrt{2}\,\pi$

(c) $\pi \ln 5$

(d) $\frac{\pi}{3} \ln 5$

(e) $\frac{\pi}{8} \ln 5$

14. If the point $(4, 4\sqrt{3}, -7)$ is given in rectangular coordinates, then its cylindrical coordinates are given by

(a) $\left(8, \frac{2\pi}{3}, 7\right)$

(b) $\left(-8, \frac{\pi}{3}, -7\right)$

(c) $\left(-8, \frac{4\pi}{3}, -7\right)$

(d) $\left(8, \frac{5\pi}{3}, 7\right)$

(e) $\left(8, \frac{2\pi}{3}, -7\right)$

15. Using four equal squares and their midpoints, the best estimate for the volume of the solid that lies above the square

$$R = \{(x, y) |\ 0 \le x \le 4, \quad 0 \le y \le 4\}$$

and below the elliptic paraboloid $f(x, y) = 68 - 2x^2 - 2y^2$ is

(a) 836

(b) 778

(c) 768

(d) 762

(e) 192

16. Evaluate the iterated integral

$$\int_0^1 \int_{x^2}^1 \sqrt{y} \sin(x\sqrt{y}) dy\ dx.$$

(a) $\cos 1$

(b) $1 - \sqrt{\sin 1}$

(c) $\sin 1$

(d) $1 - \cos 1$

(e) $1 - \sin 1$

17. If $f(x,y) = y^3 - 6xy + 8x^2$, then a local minimum of f occurs at

(a) $(0,0)$

(b) $(32,4)$

(c) $(9,3)$

(d) $\left(\frac{9}{32},\frac{3}{4}\right)$

(e) $\left(\frac{9}{2},\frac{4}{3}\right)$

18. Given that $x = 2s^2 + 3t,\ y = 3s - 2t^2,\ z = f(x,y),\ f_x(-1,1) = 7,$ $f_y(-1,1) = -3$. The value of $\frac{\partial z}{\partial s} + \frac{\partial z}{\partial t}$ at $s = 1,\ t = -1$ is

(a) 5

(b) 28

(c) 4

(d) -1

(e) 27

19. Let $\vec{u} = \langle 1, -1, 0 \rangle$ and $\vec{v} = \langle 0, 2, -1 \rangle$. If θ is the angle between $\vec{u}$ and $\vec{v}$, then $\tan\theta$ is equal to

(a) $\dfrac{\sqrt{3}}{\sqrt{2}}$

(b) $-\dfrac{2}{\sqrt{6}}$

(c) $-\dfrac{\sqrt{6}}{2}$

(d) $-\sqrt{6}$

(e) $\dfrac{\sqrt{3}}{\sqrt{5}}$

20. The length of the curve represented by

$$x = 2 + 3t, y = \cosh 3t, \quad 0 \le t \le 1$$

is equal to

(a) $\cosh 3$

(b) $\sinh 3$

(c) $\cosh 3 + \sinh 3$

(d) $(1 + \cosh 3)$

(e) $(1 + \sinh 3)$

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55	a	b	c	d	e	f
56	a	b	c	d	e	f
57	a	b	c	d	e	f
58	a	b	c	d	e	f
59	a	b	c	d	e	f
60	a	b	c	d	e	f
61	a	b	c	d	e	f
62	a	b	c	d	e	f
63	a	b	c	d	e	f
64	a	b	c	d	e	f
65	a	b	c	d	e	f
66	a	b	c	d	e	f
67	a	b	c	d	e	f
68	a	b	c	d	e	f
69	a	b	c	d	e	f
70	a	b	c	d	e	f

Q	MM	V1	V2	V3	V4
1	a	b	d	e	d
2	a	c	e	c	d
3	a	a	a	c	b
4	a	c	d	e	d
5	a	a	b	d	a
6	a	a	b	d	c
7	a	d	b	a	e
8	a	b	c	a	a
9	a	a	a	c	b
10	a	e	e	d	c
11	a	e	c	b	a
12	a	b	a	a	a
13	a	a	a	b	e
14	a	c	d	e	c
15	a	e	b	e	c
16	a	a	b	d	e
17	a	a	a	b	d
18	a	d	b	d	b
19	a	e	d	a	c
20	a	d	c	d	b

Answer Counts

V	a	b	c	d	e
1	3	1	2	9	5
2	6	3	5	5	1
3	3	4	6	3	4
4	4	1	3	5	7