

King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 201
Exam II, 2010-2011 (101)

Duration: 120 minutes

Name: _____

Id: _____

Section: _____

Answer the questions in the space provided. You must show your work or explain your solution otherwise points may be deducted. If you make an unnecessary approximation in your solution to a problem, your answer will be judged on its accuracy. Points may be deducted for poor or inappropriate approximation.

1. Write clearly.
2. Show all your steps.
3. No credits will be given to wrong steps.
4. Calculators and mobile phones are NOT allowed in this exam.

Q#	Marks	Maximum Marks
1		17
2		17
3		16
4		17
5		16
6		17
Total		100

Problem 1. (17pts)

a) Find parametric equations for the line through the point $(4, 2, 6)$ that is perpendicular to the plane

$$3x - y + z = 7,$$

and find the points where the line intersects the coordinate planes.

b) If θ is the angle between the planes

$$3(x - 1) - 2(y - 5) + 2(z + 1) = 0, \text{ and } 2x + 5(y - 1) + (z + 4) = 0,$$

find the value of $4\sec^2\theta$

Problem 2. (17pts)

a) Find an equation of the plane that contains the line

$$x = 10 + t, \quad y = 9 - 5t, \quad z = t$$

and is perpendicular to the plane $3x + 2y - 2z = 7$.

b) Identify and sketch the surface given by

$$-4x^2 + 8x + y^2 + 2y - \frac{4z^2}{9} = 7$$

Problem 3. (16pts)

a) Show that the function

$$f(\mathbf{x}, \mathbf{y}) = \begin{cases} \frac{\sin(\mathbf{x}\mathbf{y})}{\mathbf{x}^2+\mathbf{y}^2} & \text{if } (\mathbf{x}, \mathbf{y}) \neq (\mathbf{0}, \mathbf{0}) \\ \mathbf{1} & \text{if } (\mathbf{x}, \mathbf{y}) = (\mathbf{0}, \mathbf{0}) \end{cases} .$$

is not continuous at $(\mathbf{0}, \mathbf{0})$

b) Evaluate

$$\lim_{(\mathbf{x}, \mathbf{y}) \rightarrow (\mathbf{0}, \mathbf{0})} \frac{\mathbf{3} - \sqrt{\mathbf{x}^2 + \mathbf{y}^2 + \mathbf{9}}}{\mathbf{x}^2 + \mathbf{y}^2}$$

Problem 4. (17pts)

a) Find and sketch the domain of the function

$$f(\mathbf{x}, \mathbf{y}) = \sqrt{2\mathbf{x} + \mathbf{y}} \ln(\mathbf{y} - \mathbf{x}).$$

b) Use implicit differentiation to find $\frac{\partial \mathbf{z}}{\partial \mathbf{x}}$ and $\frac{\partial \mathbf{z}}{\partial \mathbf{y}}$ if

$$\mathbf{y}\mathbf{z} = \ln(\mathbf{x} + \mathbf{z}).$$

Problem 5. (16pts)

a) Find an equation of the tangent plane to the surface

$$z = xye^{-xy^2}$$

at the point $(1,1,e^{-1})$

b) Use the linearization $L(x,y)$ of the function

$$f(x,y) = \sqrt{9 - x^2 - 4y^2},$$

at $(-1,1)$ to approximate $f(-0.9, 1.1)$.

Problem 6. (17pts)

a) Find all second partial derivatives of

$$f(\mathbf{x}, \mathbf{y}) = \mathbf{x}e^{-2\mathbf{y}}$$

b) If $\mathbf{z} = \mathbf{y} + \cos(\mathbf{x}^2 - \mathbf{y}^2)$, show that

$$\mathbf{y} \frac{\partial \mathbf{z}}{\partial \mathbf{x}} + \mathbf{x} \frac{\partial \mathbf{z}}{\partial \mathbf{y}} = \mathbf{x}$$