1. (a) Eliminate the parameter to find a cartesian (rectangular) equation of the curve whose parametric equations are $x = 2\sin^2 t$ and $y = 4\sec t$, $-\dfrac{\pi}{2} < t < \dfrac{\pi}{2}$.

(b) Find an equation of the tangent line to the curve in part (a) at $t = \dfrac{\pi}{4}$.

2. (a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for the curve given by

$$x = t + t^2 \text{ and } y = t - t^2.$$

(b) Find the values of t for which the curve given in part (a) is concave down.

3. (a) Find the area of the surface obtained by rotating about the x-axis the curve given by

$$x = \sqrt{7}\cos^3\theta \text{ and } y = \sqrt{7}\sin^3\theta \quad 0 \le \theta \le \frac{\pi}{2}.$$

(b) On the same polar coordinate system, draw the unit circle centered at the origin and plot the points whose polar coordinates are $\left(2, \frac{3\pi}{4}\right)$, $\left(-\frac{3}{2}, \frac{\pi}{2}\right)$, and $\left(\frac{-1}{2}, -\frac{7\pi}{6}\right)$.

4. (a) Sketch on the same polar coordinate system the curves

$$r = \sqrt{3}\sin\theta \quad \text{and} \quad r = 1 + \cos\theta.$$

and determine polar coordinates of the intersection point(s) if any.

(b) Find the area of the region that lies inside the curve $r = \sqrt{3}\sin\theta$ and outside the curve $r = 1 + \cos\theta$.

5. (a) Find an equation of the sphere whose diameter has endpoints $(2, 1, 4)$ and $(4, 3, 10)$.

(b) Describe in words and sketch the region of the three-dimensional space represented by $z^2 = 4$.

6. (a) Given the points $A(1, 0, -1)$, $B(2, 1, 0)$, find the coordinates of the point C such that $\overrightarrow{AC}$ and $\overrightarrow{AB}$ have opposite directions and $|\overrightarrow{AC}| = 3\,|\overrightarrow{AB}|$.

(b) Consider the vectors

$$\vec{u} = \langle \sin x, 2, 1\rangle \text{ and } \vec{v} = \langle 1, \cos x, \sin x\rangle\ , \quad 0 \le x \le 2\pi.$$

Find the values of x for which the two vectors $\vec{u}$ and $\vec{v}$ are orthogonal.

7. (a) Find the volume of the parallelepiped with adjacent edges DA, DB, and DC, where $A(3,0,1), B(1,2,0), C(2,1,0), D(0,-1,0)$.

(b) Consider the vectors

$$\vec{a} = \langle -2, 1, 3\rangle \text{ and } \vec{b} = \langle 3, 3, 4\rangle.$$

If $\vec{u} = \mathbf{proj}_{\vec{a}}\vec{b}$ and $\vec{v} = \vec{b} - \vec{u}$, find the vector $\vec{w}$

$$\vec{w} = \frac{1}{2}\,\vec{u} + (5\vec{a}\cdot\vec{v})\vec{v} - \vec{a}\times(2\vec{u}).$$