

King Fahd University of Petroleum & Minerals
Department of Mathematics and Statistics

Math 101
Final Exam
Semester 101
Sunday, January 23, 2011
Net Time Allowed: 180 minutes

MASTER VERSION

1. The graph of $y = \frac{x(x-1)^2(x+1)}{x(x+2)(x-1)}$ has
- (a) one vertical, and one slant asymptotes, and two holes
 - (b) three vertical, and one slant asymptotes, and two holes
 - (c) two vertical, and one slant asymptotes and one hole
 - (d) one slant asymptote and one hole
 - (e) three vertical asymptotes and no other asymptotes

2. If $f''(x) = \frac{3}{2}\sqrt{x} + \frac{2}{3}\frac{1}{\sqrt[3]{x}}$ and $f'(0) = f(0) = 0$, then $f(1) =$

- (a) 1
- (b) 2
- (c) $\frac{5}{6}$
- (d) $\frac{1}{6}$
- (e) 3

3. The slope of the normal line to the graph of

$$f(x) = \sinh^{-1}\left(\frac{2}{3}x\right) \quad \text{at } x = -2 \text{ is}$$

- (a) $-\frac{5}{2}$
- (b) 1
- (c) $-\frac{2}{3}$
- (d) $\frac{5}{4}$
- (e) $-\frac{5}{3}$

4. The graph of the function

$$f(x) = -2x - 4 \sin x, \quad 0 \leq x \leq 2\pi$$

is decreasing on the interval(s)

- (a) $\left(0, \frac{2\pi}{3}\right)$ and $\left(\frac{4\pi}{3}, 2\pi\right)$
- (b) $\left(\frac{2\pi}{3}, \frac{4\pi}{3}\right)$
- (c) $\left(0, \frac{4\pi}{3}\right)$
- (d) $\left(\frac{\pi}{3}, \pi\right)$ and $\left(\frac{5\pi}{3}, 2\pi\right)$
- (e) $\left(\frac{2\pi}{3}, \pi\right)$ and $\left(\frac{3\pi}{2}, 2\pi\right)$

5. If $f(x) = \frac{1}{30}x^6 - \frac{1}{3}x^4 + 11x + 19$, then

- (a) f has exactly two inflection points
- (b) f has exactly three inflection points
- (c) f is concave downward on $(2, \infty)$
- (d) f is concave downward on $(-\infty, -2)$
- (e) f is concave upward on $(-2, 2)$

6. The slope of the tangent line to the curve $x - \cos(xy) = 3$ at the point $\left(3, \frac{\pi}{6}\right)$ is

- (a) $-\frac{\pi + 6}{18}$
- (b) $\frac{2\pi - 1}{9}$
- (c) $-\frac{\pi + 6}{9}$
- (d) $\frac{\pi - 2}{18}$
- (e) $-\frac{\pi + 10}{18}$

7. If $f(x) = (2x - 1)(3x - 2)(4x - 3)(5x - 4)$, then $\frac{f'(1)}{f(1)} =$

[Hint: You may use logarithmic differentiation]

(a) 14

(b) $\frac{11}{20}$

(c) 11

(d) $\frac{17}{20}$

(e) 1

8. If the area A of a circle is increasing at the rate of 2π cm²/min, then at the moment when $A = \frac{\pi}{25}$ cm² the radius of the circle is increasing at the rate of

(a) 5 cm/min

(b) 6 cm/min

(c) 7 cm/min

(d) 4 cm/min

(e) 3 cm/min

9. The shortest distance from the point $(0, 1)$ to the hyperbola $x^2 - y^2 = 3$ is equal to

(a) $\sqrt{\frac{7}{2}}$

(b) $\sqrt{\frac{15}{2}}$

(c) $\frac{2}{3}$

(d) $\frac{7}{3}$

(e) $\sqrt{\frac{3}{2}}$

10. If f' is continuous such that $f(2) = 0$ and $f'(2) = 7$, then $\lim_{x \rightarrow 0} \frac{f(2 + 3x) + f(2 + 5x)}{x} =$

(a) 56

(b) 14

(c) 60

(d) 0

(e) 7

11. Suppose that the line $y = -2x + 1$ is tangent to the curve $y = f(x)$ when $x = 1$. If Newton's method is used to locate a root of the equation $f(x) = 0$ with $x_1 = 1$, then $x_2 =$
- (a) 0.5
 - (b) 0.125
 - (c) 0.75
 - (d) 0
 - (e) 0.2
12. The difference between the absolute maximum value and the absolute minimum value of the function $f(x) = \ln(x^2 + x + 1)$ on the interval $[-1, 1]$ is equal to
- (a) $\ln 4$
 - (b) $\ln\left(\frac{7}{2}\right)$
 - (c) $\ln\left(\frac{10}{3}\right)$
 - (d) $\ln 5$
 - (e) $\ln(e + 1)$

13. If $f(x) = \log_2 \left(\frac{5}{x} \right)$, then $f^{(100)}(1) =$

(a) $\frac{99!}{\ln 2}$

(b) $\left(\frac{1}{5 \ln 2} \right)^{100} (100!)$

(c) $-\frac{99!}{(\ln 2)^{100}}$

(d) $-\left(\frac{1}{5 \ln 2} \right)^{100} (100!)$

(e) $\frac{99!}{(\ln 2)^{99}}$

14. The graph of the function $f(x) = \frac{3^x}{3^x + 1}$ has the following asymptotes

(a) two horizontal and no other asymptotes

(b) one vertical, one horizontal, and one slant asymptotes

(c) one vertical, one horizontal, and no slant asymptotes

(d) only one horizontal and no vertical asymptotes

(e) two horizontal, one slant, and no vertical asymptotes

15. If $L(x) = mx + c$ is the linearization of $f(x) = (1 - 2 \sin x)^{2/3}$ at $x = 0$, then $m + c =$

(a) $-\frac{1}{3}$

(b) 1

(c) $\frac{2}{3}$

(d) $-\frac{5}{3}$

(e) -1

16. If $f(x) = \operatorname{csch} \left(\frac{1}{2} x \right)$, then $f'(\ln 4) =$

(a) $-\frac{10}{9}$

(b) $\frac{4}{9}$

(c) $-\frac{8}{9}$

(d) $\frac{8}{9}$

(e) $-\frac{4}{9}$

17. The product of all critical numbers of the function

$$f(x) = x^{-1/5}(x - 9)^2 \text{ is}$$

- (a) -9
- (b) 0
- (c) $-\frac{9}{5}$
- (d) 3
- (e) -3

18. The sum of all numbers c that satisfy the conclusion of Rolle's Theorem for the function $f(x) = \frac{1 - \sin x}{1 + \sin x}$ on the interval $[0, \pi]$ is

- (a) $\frac{\pi}{2}$
- (b) π
- (c) $\frac{3\pi}{2}$
- (d) $\frac{3\pi}{4}$
- (e) Rolle's Theorem is not applicable

19. If $f(x) = x^4 + 4x^3$, then

- (a) $f(-3) \leq f(x)$ for all x
- (b) f has no local minimum values
- (c) f has one local maximum value
- (d) The graph of f is concave upward on $(-2, 0)$.
- (e) The graph of f has no inflection points.

20. $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{2x} + \frac{3}{x^2}\right)^x =$

- (a) \sqrt{e}
- (b) $e^{2/3}$
- (c) 1
- (d) $e^{3/2}$
- (e) $\frac{3}{2}$

21. The slope of the tangent line to the graph of $y = \tan^{-1}(x - \sqrt{9 + x^2})$ at $x = 4$ is
- (a) 0.1
 - (b) 0.02
 - (c) 0.01
 - (d) 0.2
 - (e) 0.002
22. Which one of the following statements is **TRUE** about the graph of $xy = x^2 + 9$?
- (a) The graph lies between the lines $x = 0$ and $y = x$.
 - (b) The graph has a local maximum at $(3, 6)$.
 - (c) The graph has a local minimum at $(-3, -6)$.
 - (d) The graph has no slant asymptote.
 - (e) The graph has one inflection point.

23. The number of the critical numbers of the function

$$f(x) = ||x^2 - 4| - 2| \text{ is}$$

[Hint: You may sketch the graph of f]

- (a) 7
 - (b) 5
 - (c) 3
 - (d) 1
 - (e) 0
24. Let f be a differentiable function such that $f'(x) \leq 10$ for all x in the domain of f . If $f(-1) = -2$, then the largest possible value of $f(4)$ is

- (a) 48
- (b) 12
- (c) 62
- (d) 14
- (e) 51

25. Let $f(x) = x^{-1/4}$. Using differentials with $x = 16$ to estimate a value of $f(15.84)$, the percentage error in the calculation of $f(15.84)$ is
- (a) 0.25 %
 - (b) 0.125 %
 - (c) 0.5 %
 - (d) 0.75 %
 - (e) 0.55 %
26. If the length of a rectangle is decreasing at the rate of 0.6 cm/min and the width is increasing at the rate of 0.3 cm/min, then the moment the length is 8 cm and the width is 6 cm, the angle between the diagonal and the length of the rectangle is
- (a) increasing at the rate of $\frac{3}{50}$ rad/min
 - (b) decreasing at the rate of $\frac{1}{50}$ rad/min
 - (c) increasing at the rate of $\frac{9}{50}$ rad/min
 - (d) decreasing at the rate of $\frac{9}{50}$ rad/min
 - (e) increasing at the rate of $\frac{1}{50}$ rad/min

27. If a cone is inscribed in a larger cone with height 9 m and base radius 5 m so that its vertex is at the center of the base of the larger cone, then the inner cone has maximum volume when its base radius is [Volume of a cone = $\frac{\pi}{3} r^2 h$].

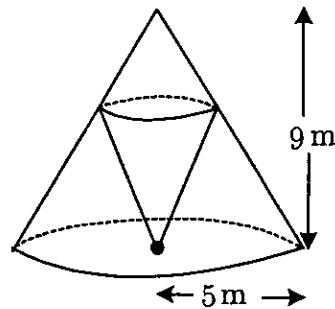
(a) $\frac{10}{3}$ m

(b) 3 m

(c) $\frac{8}{3}$ m

(d) 4 m

(e) $\frac{7}{2}$ m



28. A particle moves in a straight line and has velocity given by $v(t) = \frac{1 + 2t^2}{1 + t^2}$. If the initial displacement of the particle is $s(0) = \frac{\pi}{4}$ cm, then $s(1) =$

(a) 2 m

(b) $\frac{\pi}{2}$ m

(c) 3 m

(d) $\frac{3\pi}{4}$ m

(e) $\left(1 + \frac{\pi}{4}\right)$ m