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King Fahd University of Petroleum and Minerals
MATH-101
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Quiz 3

Name:- [illegible] **ID:-** [illegible] **Sec.:-** 25 **S.N.:-**

1. (10 points) If $f(x)=\dfrac{1+x^3}{1-x^2}$, find

i) The domain of $f(x)$

$1-x^2>0 \Rightarrow -x^2>-1 \Rightarrow x^2<1$

Domain $(-\infty,-1)(-1,1)(1,\infty)$

ii) Find the vertical asymptotes if any

vertical asyptotes are $x=-1$ $x=1$

because the denominator is zero a these values

iii) Find the horizontal asymptotes if any

$\lim_{x\to\pm\infty} \dfrac{1+x^3}{1-x^2} \Rightarrow \lim_{x\to\pm\infty} \dfrac{\frac{1}{x^2}+\frac{x^3}{x^3}}{\frac{1}{x^2}-1} = \dfrac{0+x}{0-1} = \pm\infty$

iv) Find the slant asymptotes if any

$1-x^2$ [illegible]

v) Find the interval of increase or decrease

$f'(x)=\dfrac{3x^2(1-x^2)+(1+x^3)2x}{(1-x^2)^2} = \dfrac{3x^2-3x^4+2x+2x^4}{(1-x^2)^2} = \dfrac{3x^2-x^4+2x}{(1-x^2)^2}$

increasing: $(0,1)(1,2)$ decreasing: $(-\infty,-1)(-1,0)$ $(2,\infty)$

vi) Find the local maximum and local minimum

local maximum at $x=2$

local minimum at $x=0$

vii) Find the interval of concavity

$f''(x)=\dfrac{(6x-4x^3+2)(1-x^2)^2-(3x^2-x^4+2x)2(1-x^2)(-2x)}{(1-x^2)^4}$

down $(-\infty,-1)(-1,0)$ $\cup$ $(0,1)$

viii) Find the inflection points if any

$f(0)=2$ $(0,1)$ $\Rightarrow$ inflection point

ix) Use the information from part (i)-(viii) to sketch the graph.