King Fahd University of Petroleum & Minerals
Department of Mathematics and Statistics

CODE 001 | Math 101 | CODE 001

Exam II

Semester 101

Tuesday, December 7, 2010

Net Time Allowed: 120 minutes

Name: KEY

ID: ______________ Sec: ______________.

Check that this exam has 20 questions.

Important Instructions:

1. All types of calculators, pagers or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1

1. If $f(x) = 2^{-(3x^2+x)}$, then $f'(1) =$

(a) $-\frac{3}{8}$

(b) $-\frac{1}{16}\ln 2$

(c) $-\frac{7}{16}\ln 2$

(d) $\frac{1}{16}$

(e) $\frac{1}{16}\ln 2$

$f'(x) = -2^{-(3x^2+x)} \ln 2 \, (6x+1)$

$f'(1) = 2^{-4} \ln 2 \, (-7)$

2. If the tangent line to the parabola $y = 2x^2+3x+2$ at the point (α, β) is perpendicular to the line $x + 7y = 0$, then $\alpha - \beta =$

(a) $-3/7$

(b) 3

(c) 9

(d) -6

(e) -5

$y' = 4x+3 \Rightarrow y'(\alpha) = 4\alpha + 3 = \beta$

$m_T = -\frac{1}{7} \Rightarrow m_N = 7$

$\Rightarrow 4\alpha + 3 = 7 \Rightarrow \boxed{\alpha = 1} \Rightarrow \boxed{\beta = 7}$

2

3. If $x^2 + 2xy - 3y^2 = 9$, then $\frac{dy}{dx} =$

(a) $(x+y)(3y-x)^{-1}$

(b) $(2x-y)(y-x)^{-1}$

(c) $(x+y)(6y-x)^{-1}$

(d) $2(x-3y)(3y-x)^{-1}$

(e) $2(x+y)(y-x)^{-1}$

$2x + 2y + 2xy' - 6y\,y' = 0$

$(2x - 6y)y' = -2x - 2y$

$y' = \frac{2x+2y}{6y-2x}$

$= \frac{x+y}{3y-x}$

4. The equation of motion of a particle is $S(t) = \sqrt{t} + \frac{1}{\sqrt{t}} + 19$, where S is in meters and t in minutes, then the the acceleration when the velocity is 0, is

(a) $-\frac{1}{4}\ m/\min^2$

(b) $\frac{2}{3}\ m/\min^2$

(c) $-\frac{3}{4}\ m/\min^2$

(d) $\frac{1}{2}\ m/\min^2$

(e) $\frac{3}{2}\ m/\min^2$

$s'(t) = v(t) = \frac{1}{2\sqrt{t}} - \frac{1}{2t^{3/2}} = \frac{1}{2}t^{-1/2} - \frac{1}{2}t^{-3/2}$

$= \frac{1}{2\sqrt{t}} - \frac{1}{2t\sqrt{t}} = \frac{t-1}{2t\sqrt{t}} = 0$

if $\boxed{t=1}$

$a(t) = v'(t) = -\frac{1}{4}t^{-3/2} + \frac{3}{4}t^{-5/2}$

$a(1) = -\frac{1}{4} + \frac{3}{4} = \boxed{\frac{1}{2}}\ m/min^2$

3

5. If $f(x) = g(e^{2x})$ and $g'(4) = \dfrac{1}{2}$, then $f'(\ln 2) =$

(a) 8

(b) $\frac{1}{2}$

(c) 1

(d) 4

(e) $\frac{1}{4}$

$f'(x) = g'(e^{2x}) \cdot e^{2x} \cdot 2$

$f'(\ln 2) = 2e^{2\ln 2} \cdot g'(e^{2\ln 2})$

$= 2e^{\ln 4} \cdot g'(e^{\ln 4})$

$= 2(4) \cdot g'(4)$

$= 8 \cdot \frac{1}{2} = 4$

6. If $y = \dfrac{1}{\sec x + \tan x}$, then $\dfrac{dy}{dx} = -\dfrac{\sec x \tan x + \sec^2 x}{(\sec x + \tan x)^2}$

(a) $\dfrac{-\sec x}{\sec x + \tan x}$

(b) $\dfrac{-2}{(\sec x + \tan x)^2}$

(c) $\dfrac{\tan x}{\sec x + \tan x}$

(d) $\dfrac{\tan x}{(\sec x + \tan x)^2}$

(e) $\dfrac{-2\sec x}{(\sec x + \tan x)^2}$

$= -\dfrac{\sec x(\cancel{\sec x + \tan x})}{(\sec x + \tan x)^{\cancel{2}}}$

$= -\dfrac{\sec x}{\sec x + \tan x}$

7. If $y = x\sin^{-1}x + x\cos^{-1}x$, then $x\dfrac{dy}{dx} =$ $x\left[\sin^{-1}x + \frac{x}{\sqrt{1-x^2}} + \cos^{-1}x - \frac{x}{\sqrt{1-x^2}}\right]$

(a) x^2y

(b) y

(c) 0

(d) $2x(1-x^2)^{-1/2}$

(e) xy

$= x\sin^{-1}x + x\cos^{-1}x \Rightarrow y$

8. The point on the curve $y = [\ln(2x+5)]^2$ with horizontal tangent is

(a) $(-2, 0)$

(b) $(-3, 0)$

(c) $(1, 0)$

(d) $(-4, 0)$

(e) $\left(-\frac{5}{2}, 0\right)$

$y' = 2[\ln(2x+5)]\frac{2}{2x+5}$

$= \frac{4\ln(2x+5)}{2x+5} = 0$

if $\ln(2x+5) = 0 \Rightarrow 2x+5 = 1$

$\Rightarrow \boxed{x = -2}$

$\Rightarrow y = [\ln(-4+5)]^2 = 0$

$\Rightarrow \boxed{P(-2, 0)}$

9. If the point $\left(-\frac{\pi}{4}, k\right)$ lies on the tangent line to the curve $y = \tan^{-1}(2x)$ at $x = \frac{1}{2}$, then $k =$

(a) $\frac{1}{4}$

(b) $-\frac{1}{2}$

(c) 1

(d) -1

(e) $-\frac{\pi}{2}$

$y' = \frac{2}{4x^2+1} \Rightarrow y'(\frac{1}{2}) = \frac{2}{1+1} = 1$

$y(\frac{1}{2}) = \tan^{-1}(1) = \frac{\pi}{4}$

$y - \frac{\pi}{4} = 1(x - \frac{1}{2})$

$y = x - \frac{1}{2} + \frac{\pi}{4}$

$y(-\frac{\pi}{4}) = \cancel{-\frac{\pi}{4}} - \frac{1}{2} + \cancel{\frac{\pi}{4}} = -\frac{1}{2}$

10. If $y^3 - x^3 = 1$, then $y'' =$

(a) $2x^2y^{-2}$

(b) $3xy^{-3}$

(c) x^2y^{-5}

(d) $2xy^{-5}$

(e) y^{-5}

$3y^2y' - 3x^2 = 0$

$y' = \frac{x^2}{y^2}$

$y'' = \frac{2xy^2 - 2x^2yy'}{y^4}$

$= \frac{2xy - 2x^2(\frac{x^2}{y^2})}{y^3}$

$= \frac{2xy^3 - 2x^4}{y^5}$

$= \frac{2x(y^3 - x^3)}{y^5}$

$= \frac{2x}{y^5} = 2xy^{-5}$

 6

~~11~~. If $f(x) = (1+2x)^{(1+3x)}$, then $f'(1) =$

(a) $27(\ln 3 + 4)$

(b) 108

(c) $27(9\ln 3 + 8)$

(d) $3(3\ln 3 + 8)$

(e) 54

$\ln f(x) = (1+3x)\ln(1+2x)$

$\frac{f'(x)}{f(x)} = 3\ln(1+2x) + \frac{2(1+3x)}{1+2x}$

$f'(x) = f(x)\left[3\ln(1+2x) + \frac{2(1+3x)}{1+2x}\right]$

$f'(1) = (3)^4\left[3\ln 3 + \frac{2(4)}{3}\right]$

$= 81\left[3\ln 3 + \frac{8}{3}\right]$

$= 27[9\ln 3 + 8]$

12. If $\lim_{x\to 0} \frac{\alpha \sin 2x + \beta \tan 4x}{x\cos x + 5\sin 3x} = \frac{1}{2}$, where α and β are constants,

then $\alpha + 2\beta =$

(a) 12

(b) $\frac{1}{2}$

(c) 4

(d) 6

(e) $\frac{15}{2}$

$\frac{1}{2} = \lim_{x\to 0} \frac{\alpha\frac{\sin 2x}{x} + \beta\frac{\tan 4x}{x}}{\frac{x\cos x}{x} + \frac{5\sin 3x}{x}}$

$= \lim_{x\to 0} \frac{\frac{2\alpha \sin 2x}{2x} + \frac{4\beta\tan 4x}{4x}}{\cos x + \frac{15\sin 3x}{3x}}$

$= \frac{2\alpha + 4\beta}{1+15} = \frac{2\alpha + 4\beta}{16} = \frac{1}{2}$

$\Rightarrow \frac{2(\alpha + 2\beta)}{16} = \frac{1}{2} \Rightarrow \alpha + 2\beta = \frac{16}{4} = 4$

7

13. Let $F(x) = \dfrac{[f(x)]^{\pi}}{[3+f(x)]^{e}}$ where f is a positive differentiable function. If $f(0) = f'(0) = 1$, then $F'(0) =$
[Hint: You may use logarithmic differentiation]

(a) $\dfrac{4e+\pi}{4^e}$

(b) $\dfrac{4e-\pi}{4^{e+1}}$

(c) $\dfrac{4\pi-e}{4^{e-2}}$

(d) $\dfrac{4\pi-e}{4^{e+1}}$

(e) $\dfrac{3\pi+e}{4^{e+1}}$

$$\ln F(x) = \pi \ln f(x) - e \ln(3+f(x))$$

$$\frac{F'(x)}{F(x)} = \pi \frac{f'(x)}{f(x)} - e\frac{f'(x)}{3+f(x)}$$

$$F'(x) = F(x) f'(x)\left[\frac{\pi}{f(x)} - \frac{e}{3+f(x)}\right]$$

$$F'(0) = F(0) f'(0)\left[\frac{\pi}{f(0)} - \frac{e}{3+f(0)}\right]$$

$$= \frac{[f(0)]^{\pi}}{[3+f(0)]^{e}} f'(0)\left[\frac{\pi}{f(0)} - \frac{e}{3+f(0)}\right]$$

$$= \frac{1}{4^e}\left[\pi - \frac{e}{4}\right] = \frac{4\pi - e}{4^{e+1}}$$

14. The tangent line to the curve $y = 2x^{e/2} - e^{\sin(x^2-1)+1}$ at the point $(1, 2-e)$ is parallel to the line

(a) $ex + ey = 1$

(b) $ex + y = e$

(c) the y-axis

(d) the x-axis

(e) $x + ey = e$

$$y' = 2x^{\frac{e}{2}-1}\cdot\frac{e}{2} - e^{\sin(x^2-1)+1}\, 2x\cos(x^2-1)$$

$$y'(1) = e(1) - 2e^{1}\cos(0)$$

$$= e - 2e = -e$$

$m = -e$

$ex + y = e \Rightarrow y = e \boxed{-e}x$

15. The slope of the normal line to the graph of $f(x) = \dfrac{2e^x + 1}{\sqrt{x+1}}$ at the point $(0, 3)$ is

(a) $-\dfrac{3}{2}$

(b) $-\dfrac{1}{e}$

(c) -2

(d) $\dfrac{2}{3}$

(e) e

$f(x) = (2e^x+1)(x+1)^{-1/2}$

$f'(x) = 2e^x(x+1)^{-1/2} - \frac{1}{2}(x+1)^{-3/2}(2e^x+1)$

$f'(0) = 2(1)(1) - \frac{1}{2}(1)(2+1)$

$= 2 - \frac{3}{2} = \frac{1}{2} = m_T$

$m_\perp = \dfrac{-1}{m_T} = -2$

16. If the position function S of a particle is given by the equation

$$S(t) = 2t^3 - 18t^2 + 48t + 5$$

where t is measured in seconds and S in meters, then the particle is speeding up on the time interval(s)

(a) $(0, 3)$ and $(4, \infty)$

(b) $(0, 2)$ and $(3, 4)$

(c) $(2, 4)$

(d) $(2, 3)$ and $(4, \infty)$

(e) $(1, 3)$

$v(t) = 6t^2 - 36t + 48$

$= 6(t^2 - 6t + 8)$

$= 6(t-2)(t-4) = 0$

if $t = \{2, 4\}$

$a(t) = 12t - 36$

$= 12(t-3) = 0$ if $t=3$

t		2		3		4	
$v(t)$	+	0	−		−	0	+
$a(t)$	−		−	0	+		+
	slowing		speed		slowing		speed

17. If $f(x) = \frac{1}{4}\left(\frac{x-2}{x+2}\right)$, then $f^{(55)}(-1) =$

$f'(x) = \frac{1}{4}\,\frac{\cancel{x}+2-\cancel{x}+2}{(x+2)^2} = \frac{1}{\cancel{4}}\,\frac{\cancel{4}}{(x+2)^2}$

(a) 55!

(b) $\frac{-1}{4}(55!)$

(c) 56!

(d) $\frac{1}{2}(55!)$

(e) $\frac{-1}{4}(56!)$

$f''(x) = -\frac{2}{(x+2)^3}$

$f'''(x) = \frac{2\cdot 3}{(x+2)^4}$

$f^{(4)}(x) = -\frac{2\cdot 3\cdot 4}{(x+2)^5}$

$\vdots$

$f^{(55)}(x) = (-1)^{56}\frac{55!}{(x+2)^{56}} \Rightarrow f^{(55)}(-1) = \frac{55!}{1^{56}}$

18. If $f(x) = |x+1| + 3|x-2|$, then the sum $f'(-2) + f'(1) + f'(4)$

(a) is equal to 4

(b) is equal to -1

(c) Does not exist since f is not differentiable anywhere

(d) is equal to -2

(e) Does not exist since f is discontinuous at -1 and 2

$|x+1| = \begin{cases} x+1, & x > -1 \\ -x-1, & x \le -1 \end{cases}$, $3|x-2| = \begin{cases} 3(x-2), & x > 2 \\ 3(2-x), & x \le 2 \end{cases}$

$f(x) = |x+1| + 3|x-2| = \begin{cases} 5-4x, & x \le -1 \\ 7-2x, & -1 < x \le 2 \\ 4x-5, & 2 < x \end{cases}$

$f'(x) = \begin{cases} -4, & x < -1 \\ -2, & -1 < x < 2 \\ 4, & x > 2 \end{cases} \Rightarrow$ $f'(-2) = -4$, $f'(1) = -2$ +, $f'(4) = 4$ +

$\overline{\quad -2 \quad}$

b

19. If -4 is the x-intercept of the tangent line T to the curve $y = \sqrt{x}$, then the y-intercept of T is

(a) $\frac{1}{2}$

(b) $\frac{1}{4}$

(c) 1

(d) -4

(e) -2

$y' = \frac{1}{2\sqrt{x}} \Rightarrow$ at $(a, \sqrt{a}) \Rightarrow y'(a) = \frac{1}{2\sqrt{a}}$

Slope between $(-4, 0)$ & $(a, \sqrt{a})$ is

$m = \frac{\sqrt{a}}{a+4} = \frac{1}{2\sqrt{a}} \Rightarrow 2a = a+4 \Rightarrow \boxed{a=4}$

$\Rightarrow$ Point of tangency is $P(4, 2)$

$y - 2 = \frac{1}{4}(x-4) \Rightarrow y = \frac{1}{4}x - 1 + 2$

$y = \frac{1}{4}x + 1 \Rightarrow \boxed{y = 1}$ is y-int.

20. If the function $f(x) = \begin{cases} \frac{\alpha(1-\cos 4x)}{3x^2}, & x < 0 \\ 3x + \frac{4}{\beta}, & x \geq 0 \end{cases}$

is continuous everywhere, when α and β are constants, then $\alpha\beta =$

(a) $\frac{3}{4}$

(b) $\frac{3}{2}$

(c) $-\frac{1}{2}$

(d) -3

(e) $\frac{1}{3}$

If is cont. everwhere then

$\lim_{x \to 0^-} \frac{\alpha(1-\cos 4x)}{3x^2} = 3(0) + \frac{4}{\beta} = \frac{4}{\beta}$

$\Rightarrow \frac{\alpha}{3} \lim_{x \to 0^-} \frac{(1-\cos 4x)}{x^2} = \frac{4}{\beta}$

$\Rightarrow \frac{\alpha}{3} \lim_{x \to 0^-} \frac{2\sin^2 2x}{x^2} = \frac{4}{\beta}$

$\Rightarrow \alpha\beta \lim_{x \to 0^-} \frac{\sin^2 2x}{x^2} = 6$

$\Rightarrow 4\alpha\beta \lim_{x \to 0^-} \frac{\sin^2 2x}{4x^2} = 6$

$\Rightarrow \alpha\beta \lim_{x \to 0^-} \frac{\sin^2 2x}{(2x)^2} = \frac{3}{2}$

$\Rightarrow \alpha\beta(1) = \frac{3}{2} \Rightarrow \alpha\beta = \frac{3}{2}$