King Fahd University of Petroleum & Minerals
Department of Mathematics and Statistics
MATH 101 – Calculus I
EXAM I
2009-2010 (091)

Monday, November 2, 2009 **Allowed Time: 2 Hours**

Name: _____

ID Number: _____ **Serial Number:** _____

Section Number: _____ **Instructor's Name:** _____

Instrunctions:

1. Write neatly and legibly. You may lose points for messy work.
2. **Show all your work.** No points for answers without justification.
3. **Calculators and Mobiles are not allowed**.
4. Make sure that you have 10 different problems (6 pages + cover page).

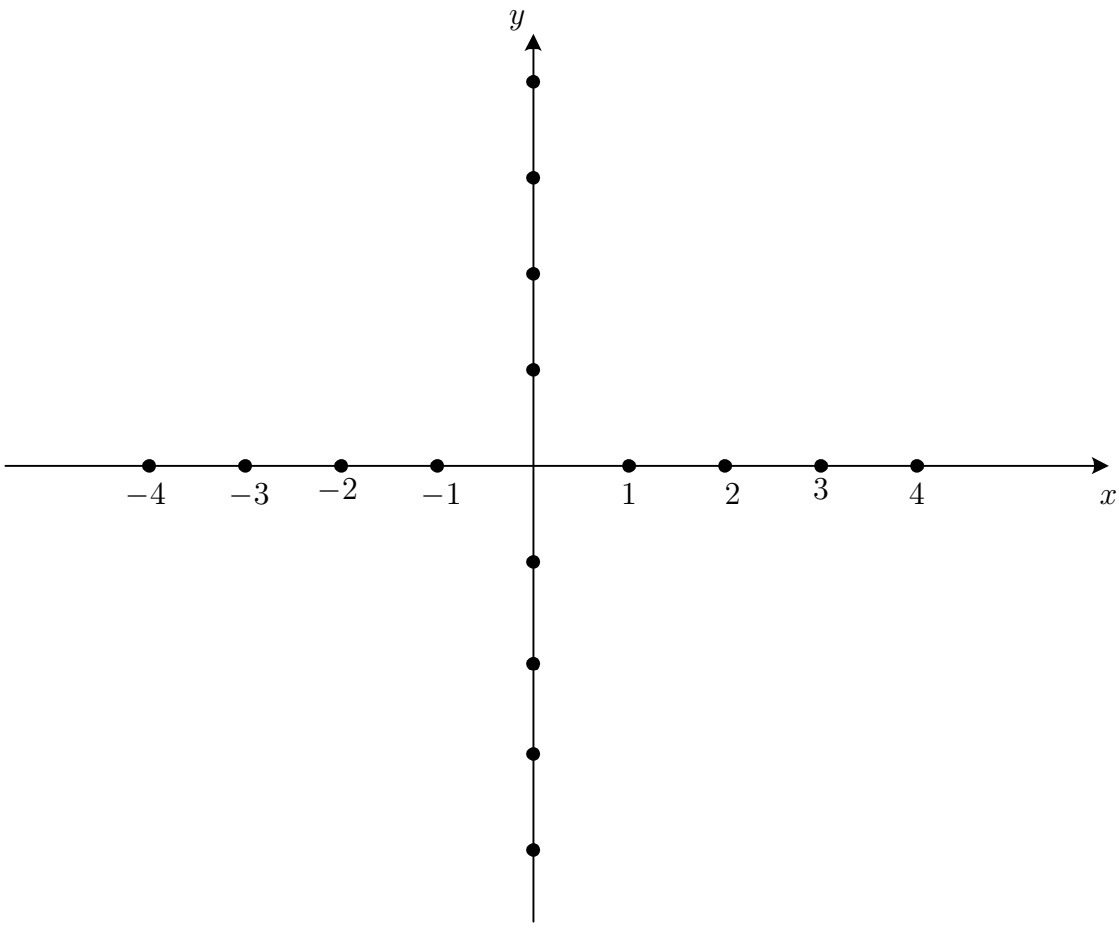
Problem No.	Points	Maxiumum Points
1		12
2		7
3(a,b,c)		18
4		12
5		6
6		10
7		7
8		12
9		12
10		4
Total:		100

1. (12-points) Sketch the graph of a function f that satisfies all of the following conditions:

   $\lim_{x\to -4^-} f(x) = -\infty\,;$ $\lim_{x\to -4^+} f(x) = \infty\,;$ $\lim_{x\to -\infty} f(x) = 0;$

   $\lim_{x\to -2} f(x) = 1\,;$ f is undefined at $-2;$

   $\lim_{x\to 1^-} f(x) = -1\,;$ $\lim_{x\to 1^+} f(x) = 2\,;$ $f(1) = 1$



2. (7-points) If $x^3 - x + 4 \le x + f(x) \le 3x^2 + 1$ for all real numbers x, then find $\lim_{x\to 1} f(x)$. (Give reasons to your steps).

3. Evaluate the limit, if it exists:

(a) (6-points) $\lim_{x\to 1/2}\left(\frac{2}{2x-1}-\frac{3}{2x^2+x-1}\right)$.

(b) (6-points) Let $f(x)=\left[\frac{1}{2}x+1\right]$ be the greatest integer less than or equal to $\frac{1}{2}x+1$. Find each of the following limits:

(i) $\lim_{x\to -2^-} f(x)$

(ii) $\lim_{x\to -2^+} f(x)$

(iii) $\lim_{x\to -2} f(x)$

(c) (6-points) $\lim_{x\to 3^-}\frac{|x^2-9|}{x-3}$.

4. (12-points) Find the horizontal asymptotes of the graph of the function

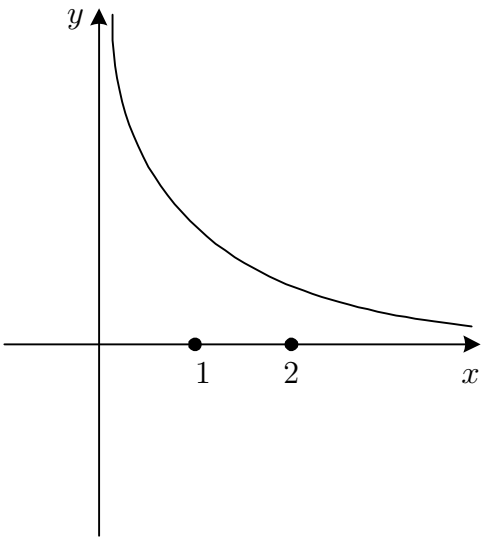
$$f(x) = \arctan \frac{\sqrt{9x^2+2}}{3x+7}\,.$$

5. (6-points) Let $f(x) = \dfrac{4-x^2}{2-x-x^2}$. Find the following limits (write the answer as a real number, ∞, or $-\infty$).

   (a) $\lim\limits_{x\to 1^-} f(x)$.

   (b) $\lim\limits_{x\to 1^+} f(x)$.

6. (10-points) Use the graph of $f(x) = \dfrac{1}{x}$ to find the largest number δ such that if $|x-1| < \delta$, then $|f(x)-1| < 0.1$. (Show your work and write your answer in simplest rational form $\dfrac{p}{q}$).



7. (7-points) Use the Intermediate Value Theorem to show that there is a root of the equation $e^{-x^2} = x$ between 0 and 1.

8. (12-points) The displacement (in meters) of a particle moving in a straight line is given by $s = \dfrac{1}{\sqrt{5-t}}$ where t is measured in seconds. Use limits to find the instantaneous velocity of the particle when $t = 1$.

9. (12 points) Find the values of a and b that make the function

$$f(x) = \begin{cases} 3 & \text{if } x = 1 \\ ax^2 - bx + 3 & \text{if } 1 < x < 2 \\ 2x - a + b & \text{if } 2 \leq x < 3 \\ 6 & \text{if } x = 3 \end{cases}$$

continuous on the closed interval $[1, 3]$. (Use limits to justify your steps)

10. (4-points) Given the function $f(x) = \dfrac{2x^2 + kx - 14}{x - 2}$, where k is a constant, find k such that $x = 2$ is a removable discontinuity of f. (Give reasons to your steps).