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Adomian decomposition method for a nonlinear heat equation with temperature dependent thermal properties

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Abstract

The solutions of nonlinear heat equation with temperature dependent diffusivity are investigated using the modified Adomian decomposition method. Analysis of the method and examples are given to show that the Adomian series solution gives an excellent approximation to the exact solution. This accuracy can be increased by increasing the number of terms in the series expansion. The Adomian solutions are presented in a number of situations of interest.

Keywords: Adomian decomposition method; temperature dependent diffusivity; nonlinear heat equation.

1. Introduction

In the classical model of the heat equation, the thermal diffusivity and thermal conductivity of the medium is assumed to be constant.* In some media such as gases, these parameters are proportional to the temperature of the medium giving rise to a more generalized nonlinear heat equation [Ozik]:

$$C(x)\frac{\partial u}{\partial t} = \lambda \frac{\partial}{\partial x}\left(ku\frac{\partial u}{\partial x}\right), \qquad (1)$$

where C is the conductivity, k is diffusivity and λ is a constant.

However, in some situation the diffusivity is proportional to u^{α}, which gives rise to a more general nonlinear heat equation:

$$C(x)\frac{\partial u}{\partial t} = \lambda \frac{\partial}{\partial x}\left(u^{\alpha}\frac{\partial u}{\partial x}\right). \qquad (2)$$

In this paper, to accommodate more general situation, we investigate the nonlinear heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(f(u)\frac{\partial u}{\partial x} \right) \qquad (3)$$

using the Adomian decomposition method. This method was presented by Adomian to solve algebraic, differential, integro-differential equations and stochastic problems [1-4]. In these papers Adomian presented the so called decomposition method in which the problem is split into linear (solvable) and nonlinear part. By assuming that the solution admits a power series representation, the nonlinear contribution to the solution is obtained in the form of "Adomian polynomials" [5]. Alternative methods of calculating Adomian polynomials have been discussed by Babolian [6] and Wazwaz [13-15]. The convergence in this regard has been established by Cherruault [7], Cherruault and Adomian [8] and, Lesnic [10]. For a detailed treatment and applications of the Adomian decomposition method one may refer to [5]. Chiu and Chen have applied the Adomian method to study fin problem with variable conductivity. We shall use the modified Adomian algorithm given by Wazwaz [14] to find the Adomian solutions to our models of non-linear heat equation with temperature dependent diffusivity.

2. The method

We consider equation (3)

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(f(u)\frac{\partial u}{\partial x} \right)$$

or

$$L_t u(x,t) = [\, f'(u)u_x^2 + f(u)u_{xx}] \qquad (4)$$

with initial condition

$$u(x,0) = g(x)$$

where L_t denotes the operator $\frac{\partial}{\partial t}$ and it is assumed that the integration (inverse operator L_t^{-1}) exists. Applying inverse operator L_t^{-1} to both sides of equation (4), yields

$$u(x,t) = u(x,0) + L_t^{-1}[(\, f'(u)u_x^2 + f(u)u_{xx})] \qquad (5)$$

The desired series solution by Adomian decomposition method is given by, cf. [1-5] for details,

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) \qquad (6)$$

and $u_1, u_2, u_3, \ldots$ are calculated from recursive relation

$$u_0 = u(x,0)$$
$$u_{n+1} = L_t^{-1}[(A_n)], \; n \geq 0 \qquad (7)$$

where A_n are the Adomian polynomials for the non-linear operator

$$F\left(u(x,t)\right) = f'(u)u_x^2 + f(u)u_{xx}.$$

The formulas that can be used to generate Adomian polynomials are discussed by Adomian in [5]. Here we employ the algorithm of Wazwaz [put reference here] to calculate Adomian polynomials, which seems quite natural and suited for implementation by software.

Example:

Consider the non-linear problem for $f(u) = u^2$ and $g(x) = \frac{x}{2}$, i.e.

$$\frac{\partial u}{\partial t} = 2uu_x^2 + u^2 u_{xx}$$
$$u(x,0) = \frac{x}{2}.$$

The exact solution is $u(x,t) = \frac{x}{2\sqrt{1-t}}$.

Setting $u(x,t) = \sum_{n=0}^{\infty} u_n(x,t)$ then the first 5 Adomian polynomials for the operator

$$F\left(u(x,t)\right) = 2uu_x^2 + u^2 u_{xx}$$

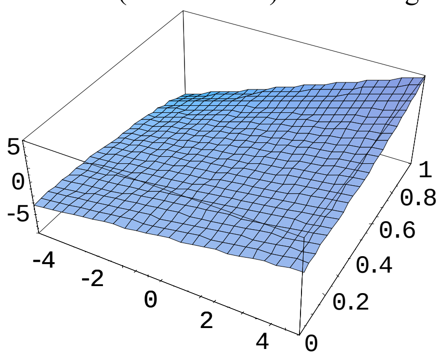
are computed as

$$A_0 = \frac{x}{4},\ A_1 = \frac{3tx}{8},\ A_2 = \frac{15t^2 x}{32},\ A_3 = \frac{35t^3 x}{64} \text{ and } A_4 = \frac{315t^4 x}{512},$$

which yield that the Adomian solution for 6 terms is

$$u(x,t) = \frac{x}{2} + \frac{tx}{4} + \frac{3t^2 x}{16} + \frac{5t^3 x}{32} + \frac{35t^4 x}{256} + \frac{63t^5 x}{512}.$$

Graph of Adomian solution (with 6 terms) for the range {x, -5, 5}, {t, 0, 1}

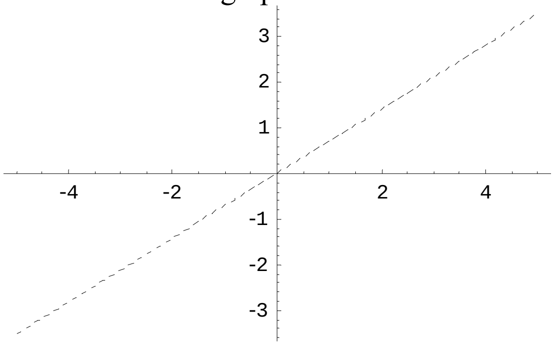


Comparison of exact and Adomian solution

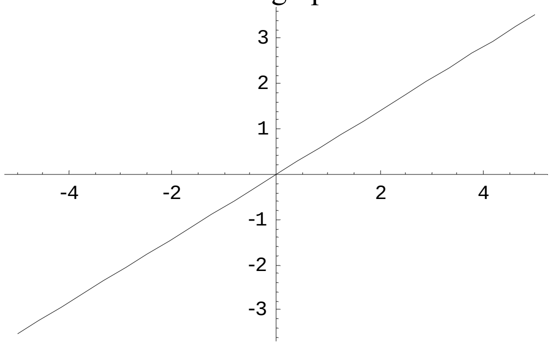
- For fixed $t = 0.5$, we have
  - Adomian solution: $u(x,0.5) = 0.703796x$
  - Exact solution: $u(x,0.5) = 0.707107x$

  and the graphs of the two solution for $t = 0.5$ are

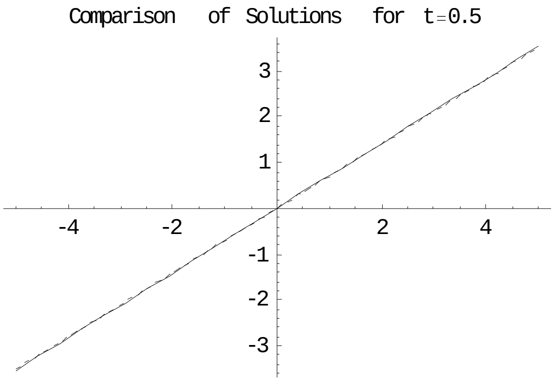
Adomian graph for -5< x <5



Exact graph for -5< x <5



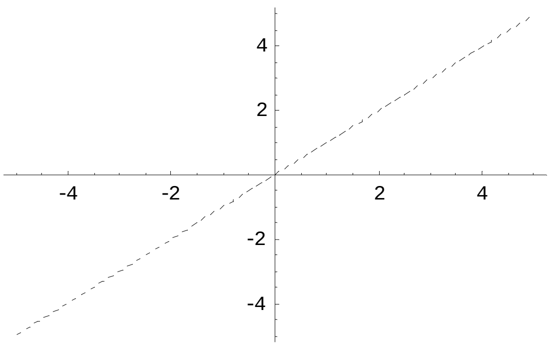
A comparison of both the graphs is



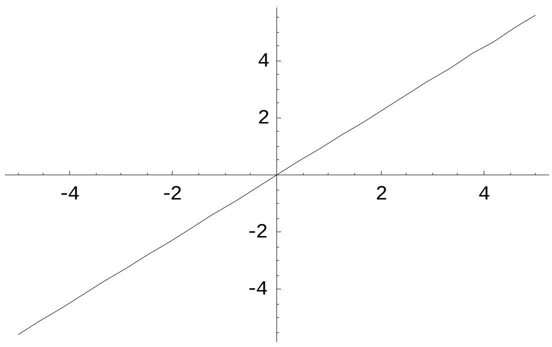
- For fixed t = 0.8, we have
  - Adomian solution: $u(x,0.8) = 0.99632x$
  - Exact solution: $u(x,0.8) = 1.11803x$

  and the graphs of the two solution for t = 0.8 are

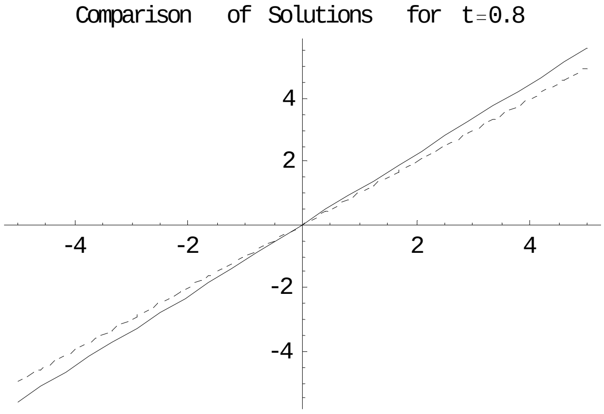
Adomian graph for -5< x <5



Exact graph for -5< x <5



A comparison of both the graphs is given below.



3. Applications and results

In this section we study different classes of the non-linear heat equation according to different types of diffusion functions $f(u)$ and different forms of initial conditions $g(x)$. The Adomian solution in all the cases below can be constructed as illustrated in the Section 2. All the solutions presented in this Section are 6-terms solutions i.e. obtained using 5 Adomian polynomials; however more terms can be generated by following the recursive process explained above. Since the expressions for the Adomian solutions, in general, are too long to put in the paper, the solutions for all the cases can be obtained from authors as Mathematica file. For cases where the expressions are not too long, the Adomian solutions are included in the paper.

3.1 Case 1

We consider the nonlinear heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(f(u)\frac{\partial u}{\partial x}\right), \qquad f(u) = u^m$$

$$u(x,0) = g(x)$$

(1-a) $f(u) = u^m$, $\quad g(x) = ax^2 + bx + c$

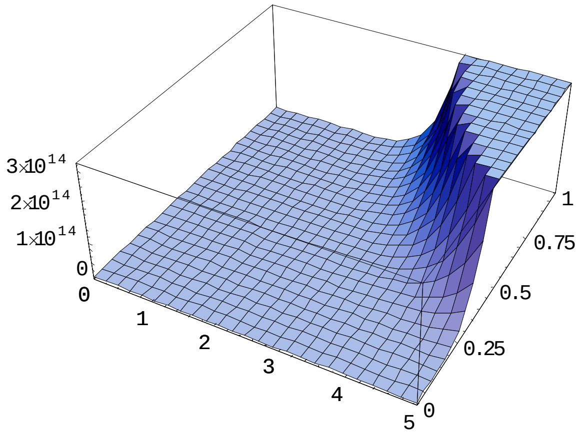
The Adomian solution $u(x,t)$ for general a, b, c and m can be obtained from authors as Mathematica file. Some particular cases for a, b, c and m are considered as follows.

(i) $a = b = c = 1$ and $m = 2$

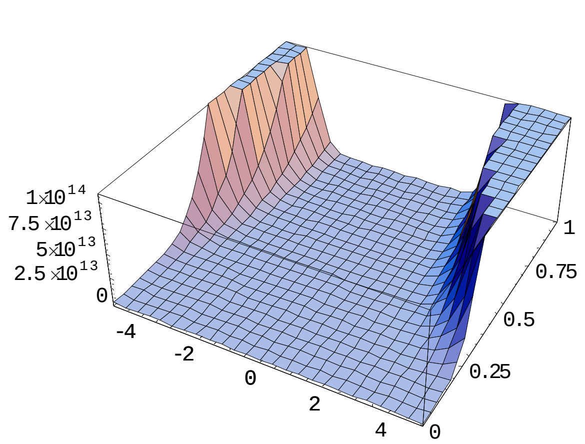
The Adomian solution is

$$
\begin{aligned}
u(x,t) = {} & 1 + x + x^2 + 2t^2 (1 + x(1 + x)) \left(24 + 45x + 185x^2 + 4x\left(25 + 70x^2\right) + \right. \\
& \left. 4\left(2 + 25x^2 + 35x^4\right)\right) + \frac{1}{3} t^3 (1 + x(1 + x)) \left(60 + 1860x + 8x\left(1860 + 7995x^2\right) + \right. \\
& 2\left(570 + 8370x^2\right) + 8x\left(1665 + 10770x^2 + 12825x^4\right) + 4\left(720 + 14490x^2 + 29370x^4\right) + \\
& \left. 8\left(75 + 1665x^2 + 5385x^4 + 4275x^6\right)\right) + \frac{1}{3} t^4 (1 + x(1 + x)) \left(8160 + 11310x + 180390x^2 + \right. \\
& 4x\left(50610 + 318390x^2\right) + 4\left(13380 + 414000x^2 + 1172820x^4\right) + \\
& 4x\left(160950 + 1507380x^2 + 2417550x^4\right) + 16x\left(20370 + 222060x^2 + 585450x^4 + 429000x^6\right) + \\
& 4\left(17970 + 605070x^2 + 2705190x^4 + 2807850x^6\right) + \\
& \left. 16\left(600 + 20370x^2 + 111030x^4 + 195150x^6 + 107250x^8\right)\right) + \\
& \frac{1}{60} t^5 (1 + x(1 + x)) \left(15120 + 1275120x + 2\left(486360 + 16082040x^2\right) + \right. \\
& 8x\left(5172000 + 45672540x^2\right) + 4\left(3226560 + 145378920x^2 + 558401640x^4\right) + \\
& 8x\left(36485460 + 477387720x^2 + 1007253300x^4\right) + \\
& 64x\left(8126460 + 123096840x^2 + 424022700x^4 + 390053400x^6\right) + \\
& 8\left(5708700 + 279944820x^2 + 1685856660x^4 + 2244119100x^6\right) + \\
& 32x\left(5402520 + 86621760x^2 + 373392000x^4 + 591280800x^6 + 309309000x^8\right) + \\
& 16\left(2417760 + 119127600x^2 + 868513680x^4 + 1945018800x^6 + 1317980400x^8\right) + \\
& \left. 32\left(113400 + 5402520x^2 + 43310880x^4 + 124464000x^6 + 147820200x^8 + 61861800x^{10}\right)\right) + \\
& t(1 + x(1 + x))(2 + 2(1 + 5x(1 + x)))
\end{aligned}
$$

Graph for the range $\{x, 0, 5\}$, $\{t, 0, 1\}$

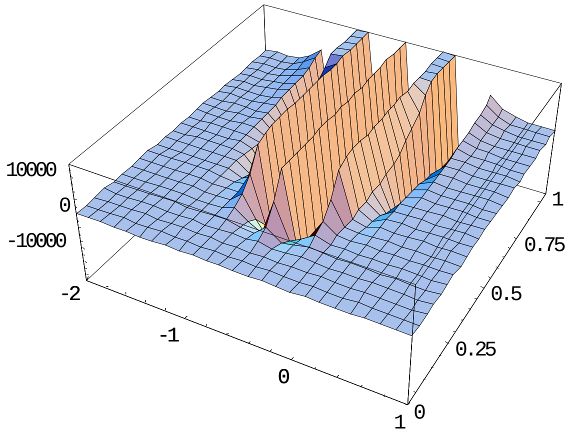


Graph for the range $\{x, -5, 5\}$, $\{t, 0, 1\}$



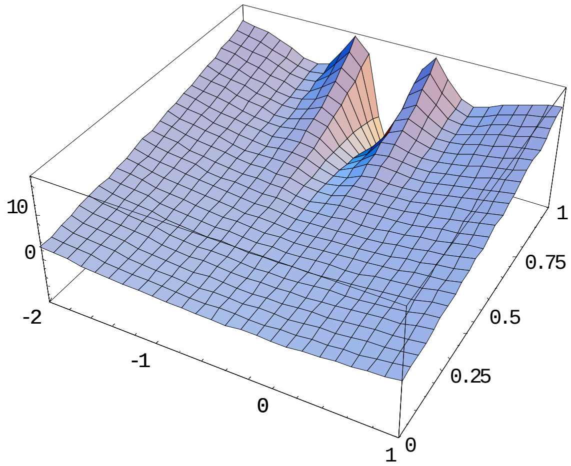
(ii) $a = b = c = 1$ and $m = -2$

Graph of Adomian solution for the range $\{x, -2, 1\}, \{t, 0, 1\}$



(iii) $a = b = c = 1$ and $m = \frac{1}{2}$

Graph of Adomian solution for the range $\{x, -2, 1\}, \{t, 0, 1\}$



(1-b) $f(u) = u^m, \quad g(x) = e^{ax}$

The Adomian solution for general a, m is

$$
\begin{aligned}
u(x,t) = {} & e^{ax} + a^2 (e^{ax})^{1+m} (1+m)\, t + \\
& \frac{1}{6} a^6 (e^{ax})^{1+3m} (1+m)(1+3m)^2 (1+5m(1+m))\, t^3 + \\
& \frac{1}{24} a^8 (e^{ax})^{1+4m} (1+m)(1+4m)^2 (1+m(13+2m(29+m(50+29m))))\, t^4 + \\
& \frac{1}{120} a^{10} (e^{ax})^{1+5m} (1+m)(1+5m)^2 \\
& (1+m(24+m(226+m(1044+m(2432+m(2736+1181m))))))\, t^5 + \\
& \frac{1}{2} a^4 (e^{ax})^{1+2m} (1+m)(t+2mt)^2
\end{aligned}
$$

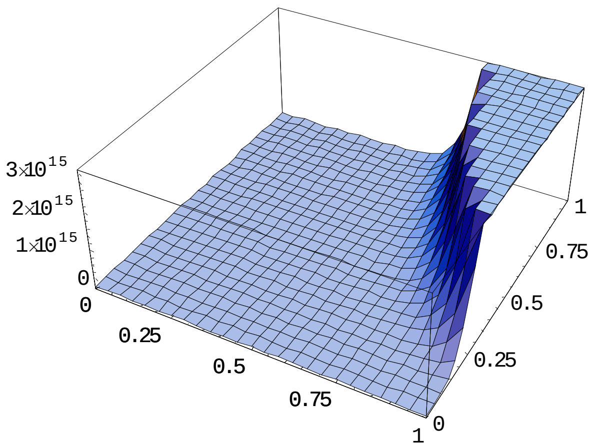
Graphs for particular cases of a, m are given below.

(i) $a = 2$ and $m = 2$

The Adomian solution becomes

$$u(x,t) = e^{2x} + 12e^{6x}t + 600e^{10x}t^2 + 48608e^{14x}t^3 + 5150304e^{18x}t^4 + \frac{3273435264}{5}e^{22x}t^5$$

Graph of Adomian solution for the range $\{x, 0, 1\}, \{t, 0, 1\}$

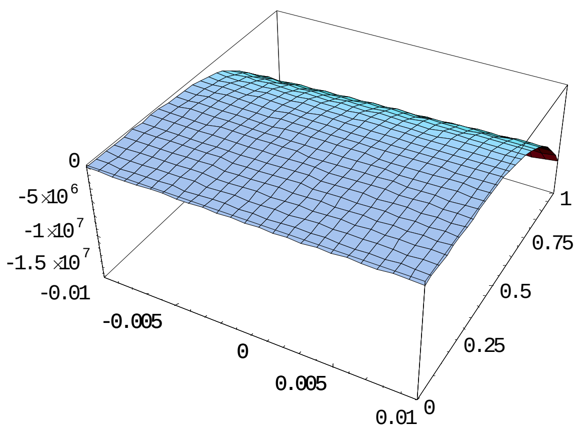


(ii) $a = 2$ and $m = -2$

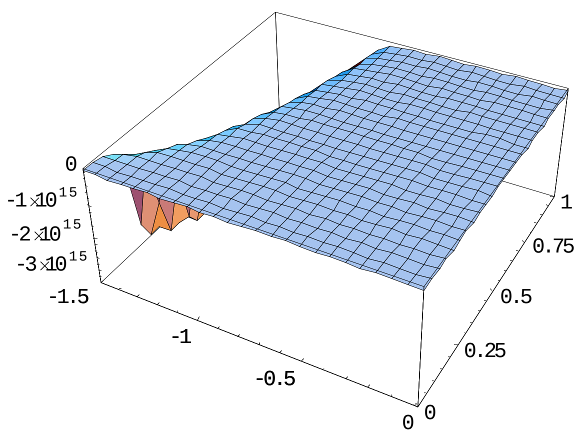
The Adomian solution is

$$u(x,t) = e^{2x} - 4e^{-2x}t - 72e^{-6x}t^2 - \frac{8800}{3}e^{-10x}t^3 - \frac{525280}{3}e^{-14x}t^4 - \frac{67215744}{5}e^{-18x}t^5$$

Graph for the range $\{x, -.01, .01\}, \{t, 0, 1\}$



Graph for the range $\{x, -1.5, .01\}, \{t, 0, 1\}$

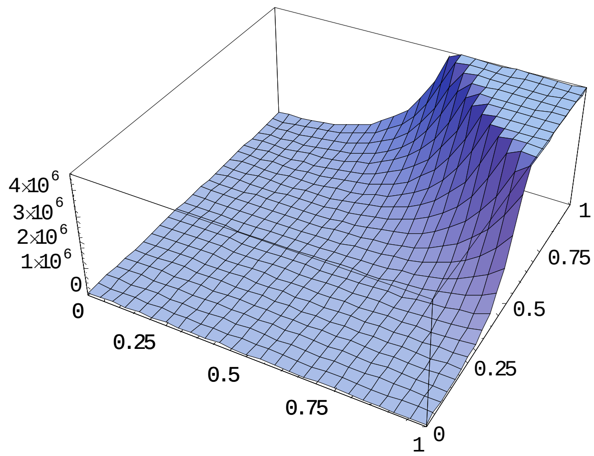


(iii) $a = 2$ and $m = \frac{1}{2}$

The Adomian solution is

$$u(x, t) = e^{2x} + 6\left(e^{2x}\right)^{3/2} t + 48 e^{4x} t^2 + 475\left(e^{2x}\right)^{5/2} t^3 + 5490 e^{6x} t^4 + \frac{1429869}{20}\left(e^{2x}\right)^{7/2} t^5$$

Graph of Adomian solution for the range $\{x, 0, 1\}, \{t, 0, 1\}$

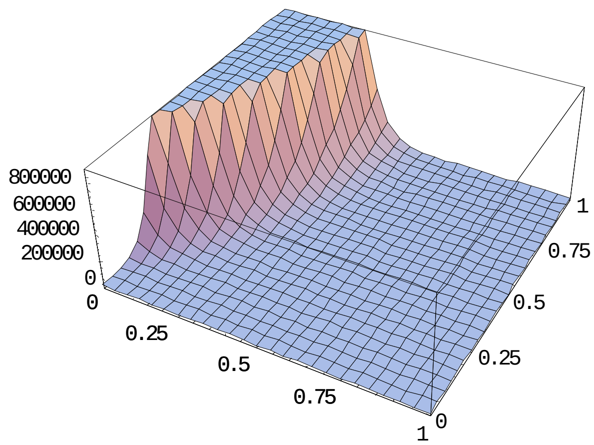


(iv) $a = -2$ and $m = 2$

The Adomian solution becomes

$$u(x, t) = e^{-2x} + 12 e^{-6x} t + 600 e^{-10x} t^2 + 48608 e^{-14x} t^3 + 5150304 e^{-18x} t^4 + \frac{3273435264}{5} e^{-22x} t^5$$

Graph of Adomian solution for the range $\{x, 0, 1\}, \{t, 0, 1\}$

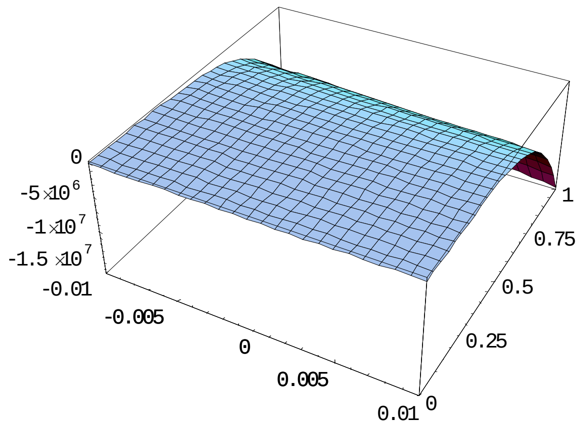


(v) $a = -2$ and $m = -2$

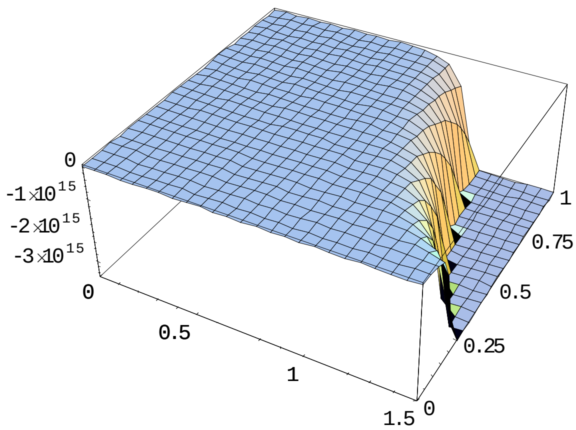
The Adomian solution is

$$u(x,t) = e^{-2x} - 4e^{2x}t - 72e^{6x}t^2 - \frac{8800}{3}e^{10x}t^3 - \frac{525280}{3}e^{14x}t^4 - \frac{67215744}{5}e^{18x}t^5$$

Graph for the range {*x*, -.01, .01}, {*t*, 0, 1}



Graph for the range {*x*, 0, 1.5}, {*t*, 0, 1}

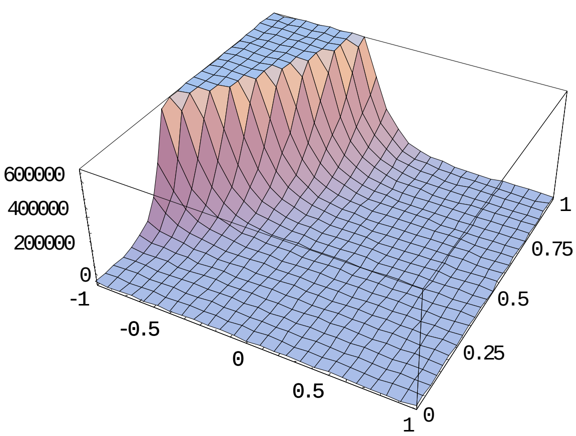


(vi) $a = -2$ and $m = \frac{1}{2}$

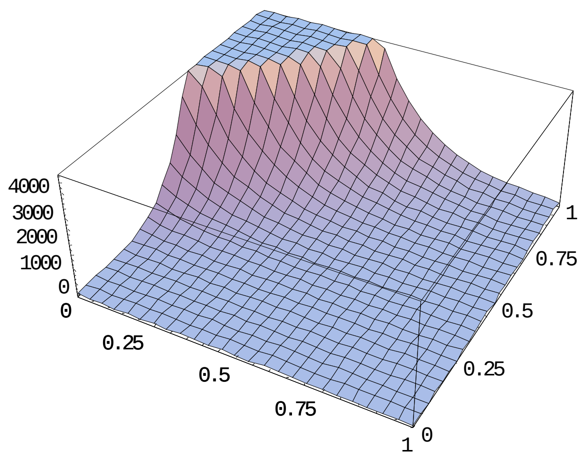
The Adomian solution is

$$u(x,t) = e^{-2x} + 6\left(e^{-2x}\right)^{3/2}t + 48e^{-4x}t^2 + 475\left(e^{-2x}\right)^{5/2}t^3 + 5490e^{-6x}t^4 + \frac{1429869}{20}\left(e^{-2x}\right)^{7/2}t^5$$

Graph for the range {*x*, -1, 1}, {*t*, 0, 1}



Graph for the range {*x*, 0, 1}, {*t*, 0, 1}



(1-c) $f(u) = u^m$, $g(x) = e^{-ax^2}$

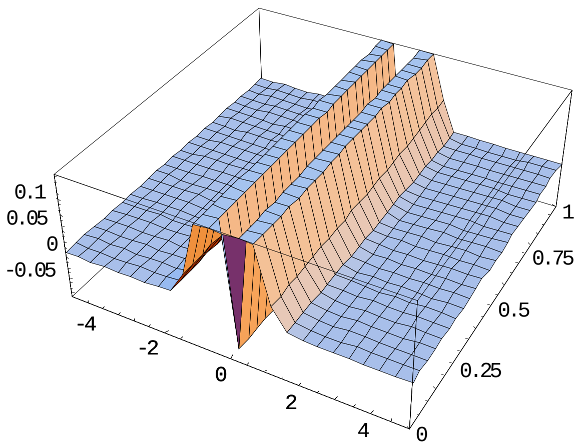
The Adomian solution for general a, m can be obtained from authors as Mathematica file. Some particular cases are considered below.

(i) $a = 2$ and $m = 2$

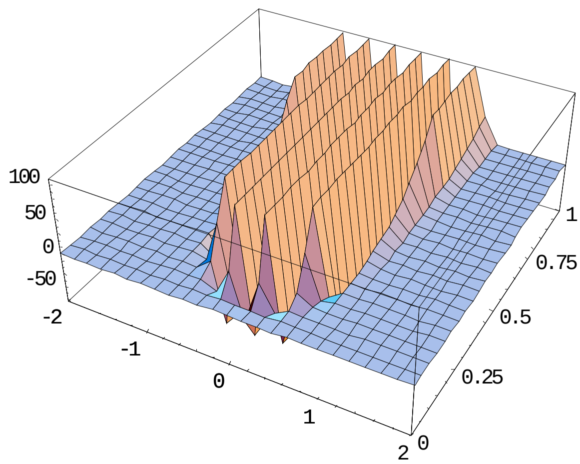
The Adomian solution is

$$u(x,t) = e^{-2x^2} + 4e^{-6x^2} t(-1+12x^2) + 8e^{-10x^2} t^2 (11 - 400x^2 + 1200x^4) + \frac{32}{3} e^{-14x^2} t^3 (-315 + 22692x^2 - 181552x^4 + 291648x^6) + \frac{32}{3} e^{-18x^2} t^4 (16425 - 1947360x^2 + 28962720x^4 - 115402752x^6 + 123607296x^8) + \frac{128}{15} e^{-22x^2} t^5 (-1326840 + 233242200x^2 - 5491343520x^4 + 38961513344x^6 - 99063148800x^8 + 78562446336x^{10} + 945(-1+20x^2))$$

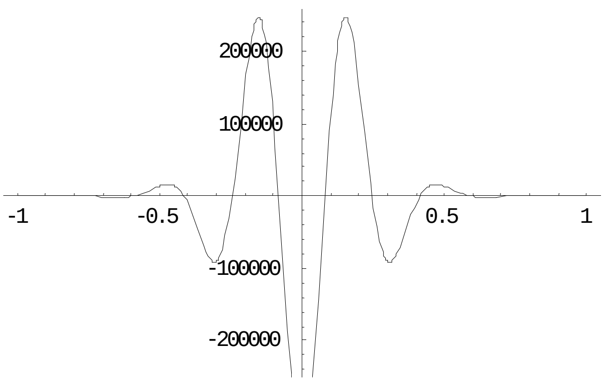
Graph for the range {*x*, -5, 5}, {*t*, 0, 1}



Graph for the range {*x*, -2, 2}, {*t*, 0, 1}



Graph for fixed t=0.5 for the range {*x*, -1, 1}

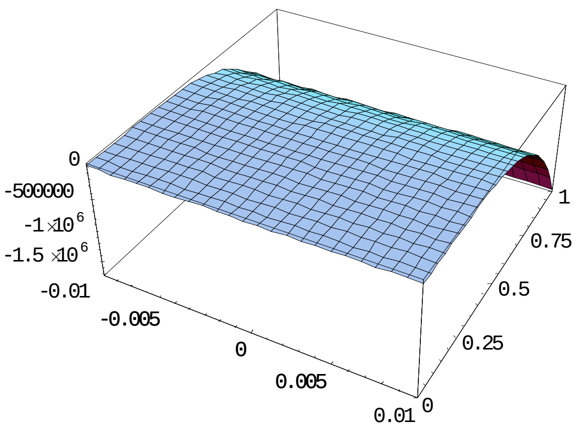


(ii) $a = 2$ and $m = -2$

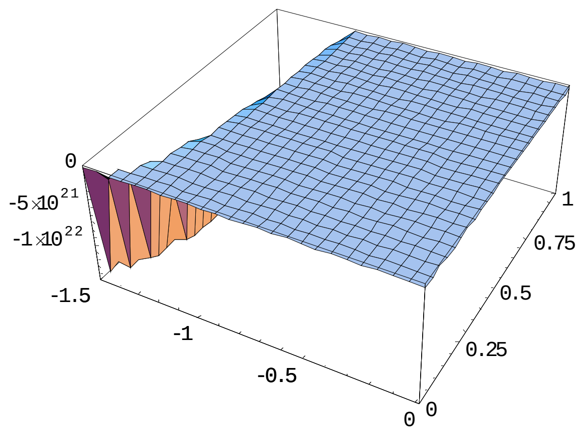
The Adomian solution is

$$u(x,t) = e^{-2x^2} + 4e^{2x^2} t(-1-4x^2) + 8e^{6x^2} t^2 (-5-96x^2-144x^4) + \frac{32}{3} e^{10x^2} t^3 (-91-4028x^2-19120x^4-17600x^6) + \frac{32}{3} e^{14x^2} t^4 (-3287-260480x^2-2523104x^4-6375936x^6-4202240x^8) + \frac{128}{15} e^{18x^2} t^5 (-191704-23954712x^2-390296736x^4-1877037696x^6-3158528256x^8-1613177856x^{10}+945(-1+20x^2))$$

Graph for the range {*x*, -.01, .01}, {*t*, 0, 1}



Graph for the range {*x*, -1.5, .01},{*t*, 0, 1}

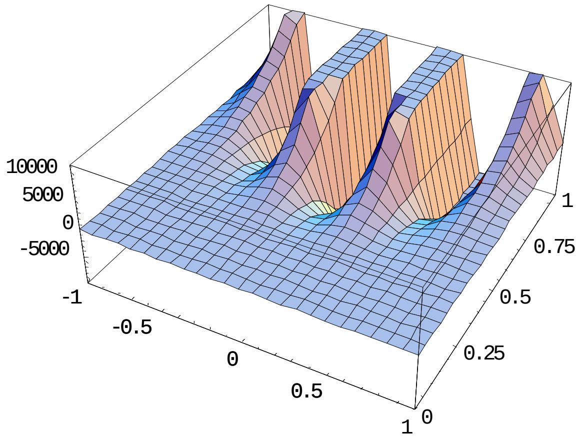


(iii) $a = 2$ and $m = \frac{1}{2}$

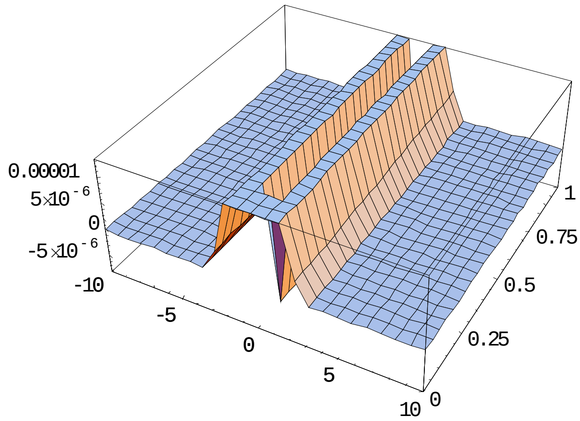
The Adomian solution is

$$u(x,t) = e^{-2x^2} + 4\left(e^{-2x^2}\right)^{3/2} t(-1+6x^2) + 8e^{-4x^2} t^2 (5-76x^2+96x^4) + \frac{32}{3}\left(e^{-2x^2}\right)^{5/2} t^3 \left(-\frac{219}{4} + \frac{3009x^2}{2} - 4615x^4 + 2850x^6\right) + \frac{32}{3} e^{-6x^2} t^4 \left(\frac{2031}{2} - 43365x^2 + 233406x^4 - 337716x^6 + 131760x^8\right) + \frac{128}{15}\left(e^{-2x^2}\right)^{7/2} t^5 \left(-\frac{433389}{16} + \frac{13476579x^2}{8} - \frac{27882147x^4}{2} + 34527269x^6 - 30761241x^8 + 8579214x^{10} + 945(-1+20x^2)\right)$$

Graph for the range $\{x, -1, 1\}, \{t, 0, 1\}$



Graph for the range $\{x, -10, 10\}, \{t, 0, 1\}$



(1-d) $f(u) = u^m, \quad g(x) = \mathrm{sech}^2 x$

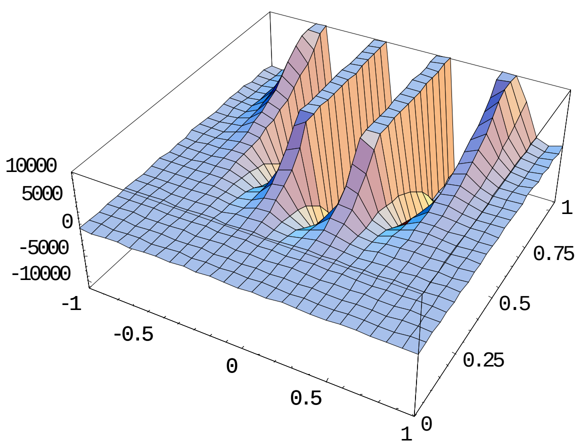
The Adomian solution for general m can be obtained from authors as Mathematica file. Some particular cases are considered below.

(i) $m = 2$

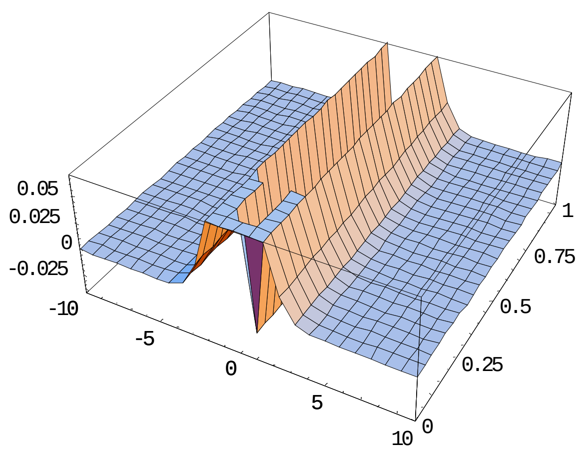
The Adomian solution is

$$
\begin{aligned}
u(x, t) = {} & \mathrm{Sech}[x]^2 + 2t(-4 + 3\,\mathrm{Cosh}[2x])\,\mathrm{Sech}[x]^8 + \\
& 3t^2(161 - 178\,\mathrm{Cosh}[2x] + 25\,\mathrm{Cosh}[4x])\,\mathrm{Sech}[x]^{14} + \\
& t^3(-54900 + 71641\,\mathrm{Cosh}[2x] - 18772\,\mathrm{Cosh}[4x] + 1519\,\mathrm{Cosh}[6x])\,\mathrm{Sech}[x]^{20} + \\
& \frac{1}{4}t^4 \\
& (35318621 - 50550350\,\mathrm{Cosh}[2x] + 18047504\,\mathrm{Cosh}[4x] - \\
& 2916178\,\mathrm{Cosh}[6x] + 160947\,\mathrm{Cosh}[8x])\,\mathrm{Sech}[x]^{26} + \\
& \frac{1}{20}t^5(-35893153056 + 54495231330\,\mathrm{Cosh}[2x] - 23506173696\,\mathrm{Cosh}[4x] + \\
& 5488700877\,\mathrm{Cosh}[6x] - 621401568\,\mathrm{Cosh}[8x] + 25573713\,\mathrm{Cosh}[10x])\,\mathrm{Sech}[x]^{32}
\end{aligned}
$$

Graph for the range $\{x, -1, 1\}, \{t, 0, 1\}$



Graph for the range $\{x, -10, 10\}, \{t, 0, 1\}$

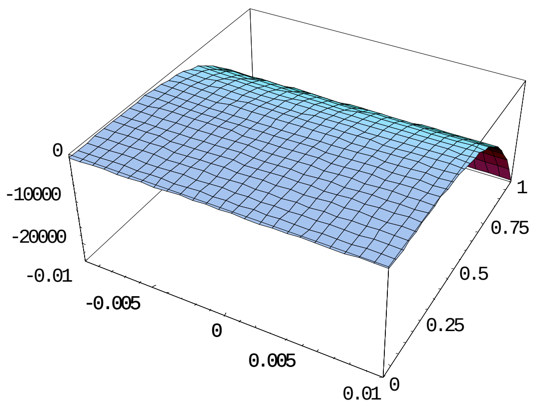


(ii) $m = -2$

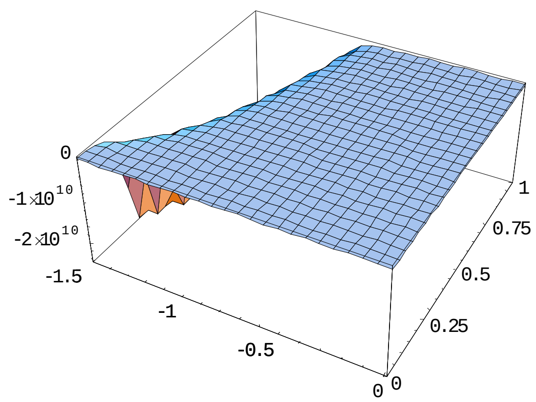
The Adomian solution is

$$u(x, t) = -2\,t\,\text{Cosh}[2x] - t^2\,\text{Cosh}[x]^2\,(1 - 2\,\text{Cosh}[2x] + 9\,\text{Cosh}[4x]) - \frac{1}{3}\,t^3\,\text{Cosh}[x]^4\,(-44 + 85\,\text{Cosh}[2x] - 76\,\text{Cosh}[4x] + 275\,\text{Cosh}[6x]) - \frac{1}{12}\,t^4\,\text{Cosh}[x]^6\,(2865 - 5862\,\text{Cosh}[2x] + 5968\,\text{Cosh}[4x] - 5178\,\text{Cosh}[6x] + 16415\,\text{Cosh}[8x]) - \frac{1}{60}\,t^5\,\text{Cosh}[x]^8\,(-303864 + 606738\,\text{Cosh}[2x] - 616768\,\text{Cosh}[4x] + 638373\,\text{Cosh}[6x] - 544328\,\text{Cosh}[8x] + 1575369\,\text{Cosh}[10x]) + \text{Sech}[x]^2$$

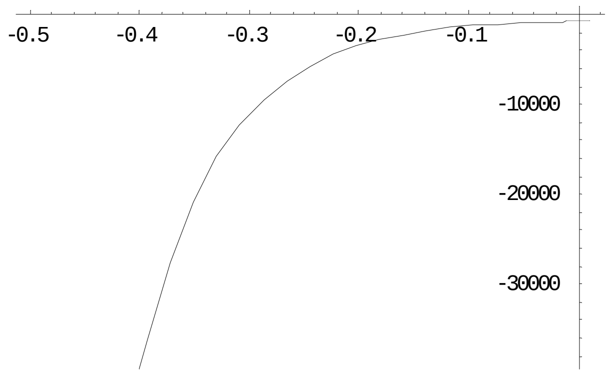
Graph for the range {*x*, -.01, .01}, {*t*, 0, 1}



Graph for the range {*x*, -1.5, .01},{*t*, 0, 1}



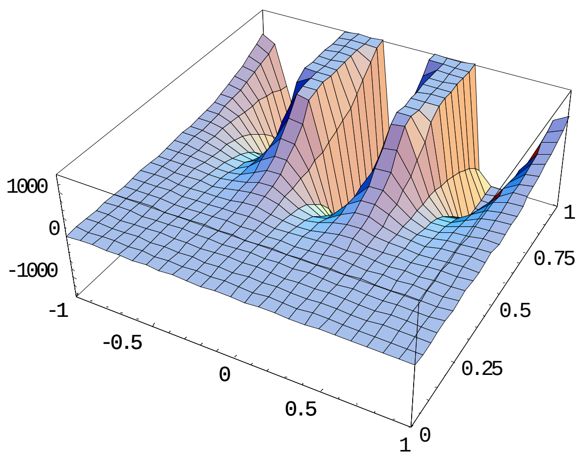
Graph for fixed t=0.5 for the range {*x*, -0.5, 0.01}



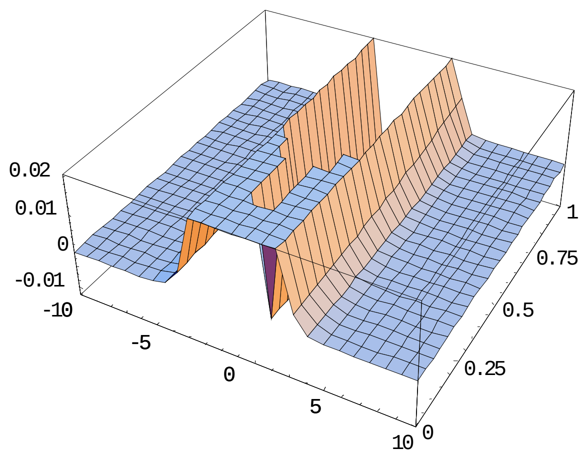
(iii) $m = \frac{1}{2}$

The Adomian solution is

$$u(x,t) = \mathrm{Sech}[x]^2 + \frac{3}{2} t^2 (56 - 52\,\mathrm{Cosh}[2x] + 4\,\mathrm{Cosh}[4x])\,\mathrm{Sech}[x]^8 + \frac{1}{8} t^4 \left(\frac{5889415}{8} - \frac{3750383}{4}\mathrm{Cosh}[2x] + 232028\,\mathrm{Cosh}[4x] - \frac{76417}{4}\mathrm{Cosh}[6x] + \frac{2745}{8}\mathrm{Cosh}[8x]\right)\mathrm{Sech}[x]^{14} + 2t\left(-\frac{5}{2} + \frac{3}{2}\mathrm{Cosh}[2x]\right)\mathrm{Sech}[x]^4 \sqrt{\mathrm{Sech}[x]^2} + \frac{1}{2} t^3 \left(-\frac{37917}{8} + \frac{86005}{16}\mathrm{Cosh}[2x] - \frac{7163}{8}\mathrm{Cosh}[4x] + \frac{475}{16}\mathrm{Cosh}[6x]\right)\mathrm{Sech}[x]^8 \left(\mathrm{Sech}[x]^2\right)^{3/2} + \frac{1}{40} t^5 \left(-\frac{22986251157}{128} + \frac{31585649589}{128}\mathrm{Cosh}[2x] - \frac{2501116101}{32}\mathrm{Cosh}[4x] + \frac{2695647273}{256}\mathrm{Cosh}[6x] - \frac{64605399}{128}\mathrm{Cosh}[8x] + \frac{1429869}{256}\mathrm{Cosh}[10x]\right)\mathrm{Sech}[x]^{12}\left(\mathrm{Sech}[x]^2\right)^{5/2}$$



Graph for the range {*x*, -1, 1}, {*t*, 0, 1}



Graph for the range {*x*, -10, 10},{*t*, 0, 1}

3.2 Case 2

We consider the nonlinear heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(f(u)\frac{\partial u}{\partial x}\right), \qquad f(u) = e^u$$

$$u(x,0) = g(x)$$

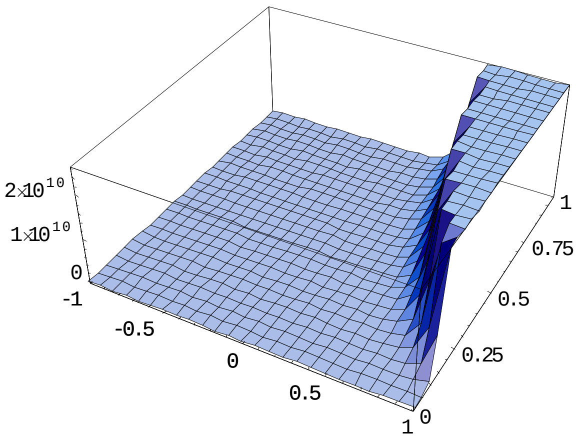
(2-a) $f(u) = e^u$, $\quad g(x) = ax^2 + bx + c$

The Adomian solution $u(x,t)$ for general a, b, c is given by

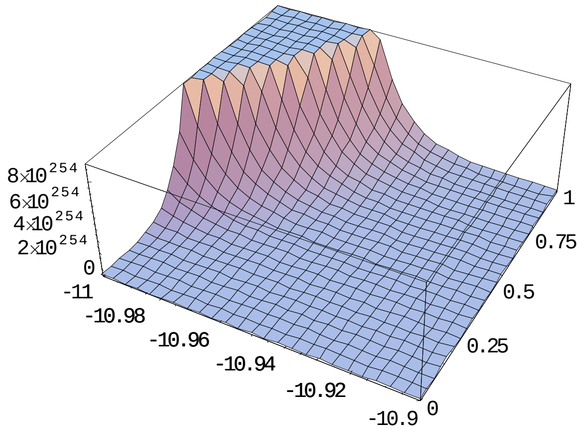
$$u(x,t) = c + bx + ax^2 + 2e^{2(c+x(b+ax))}t^2\left(b^4 + 16a^4x^4 + 4a^2(1+bx)(1+6bx) + ab^2(7+8bx) + 4a^3x^2(7+8bx)\right) + e^{c+x(b+ax)}t\left(b^2 + 2a(1+2x(b+ax))\right) + \frac{1}{6}e^{3(c+x(b+ax))}t^3\left(45b^6 + 2880a^6x^6 + 18ab^4(31+30bx) + 288a^5x^4(31+30bx) + 12a^2b^2(115+3bx(124+75bx)) + 48a^4x^2(115+3bx(124+75bx)) + 8a^3(47+6bx(115+3bx(93+50bx)))\right) + \frac{2}{3}e^{4(c+x(b+ax))}t^4\left(58b^8 + 14848a^8x^8 + ab^6(1049+928bx) + 64a^7x^6(1049+928bx) + a^2b^4(5067+4bx(3147+1624bx)) + 16a^6x^4(5067+4bx(3147+1624bx)) + 4a^3b^2(1629+bx(10134+bx(15735+6496bx))) + 16a^5x^2(1629+bx(10134+bx(15735+6496bx))) + 4a^4(261+2bx(3258+bx(15201+20bx(1049+406bx))))\right) + \frac{1}{120}e^{5(c+x(b+ax))}t^5\left(29525b^{10} + 30233600a^{10}x^{10} + 10ab^8(70877+59050bx) + 2560a^9x^8(70877+59050bx) + 4a^2b^6(1318286+5bx(567016+265725bx)) + 256a^8x^6(1318286+5bx(567016+265725bx)) + 16a^3b^4(873375+2bx(1977429+5bx(496139+177150bx))) + 256a^7x^4(873375+2bx(1977429+5bx(496139+177150bx))) + 80a^4b^2(138693+2bx(698700+bx(1977429+7bx(283508+88575bx)))) + 320a^6x^2(138693+2bx(698700+bx(1977429+7bx(283508+88575bx)))) + 32a^5(36669+10bx(138693+bx(1048050+bx(2636572+35bx(70877+21258bx)))))\right)$$

Some properties of the solution for the case $a = b = c = 1$ are shown by the graphs below.

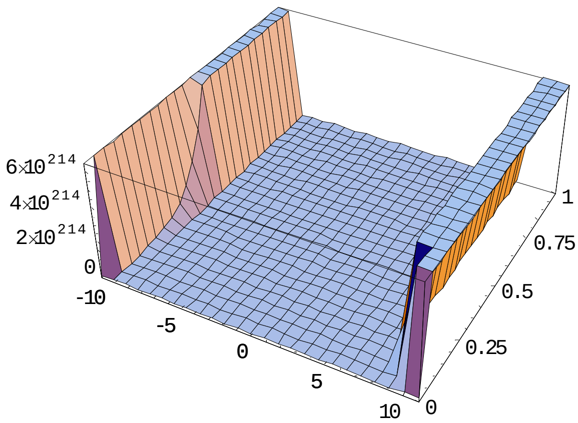
Graph for the range {*x*, -1, 1}, {*t*, 0, 1}



Graph for the range {*x*, -11, -10.9},{*t*, 0, 1}



Graph for the range {*x*, -1, 1}, {*t*, 0, 1}



(2-b) $f(u) = e^u, \qquad g(x) = e^{ax}$

The Adomian solution for general '*a*' is given by

$$u(x, t) = e^{ax} + a^2 e^{e^{ax}+ax} (1 + e^{ax})\, t + \frac{1}{2} a^4 e^{2e^{ax}+ax} \left(1 + 10\, e^{ax} + 14\, e^{2ax} + 4\, e^{3ax}\right) t^2 +$$

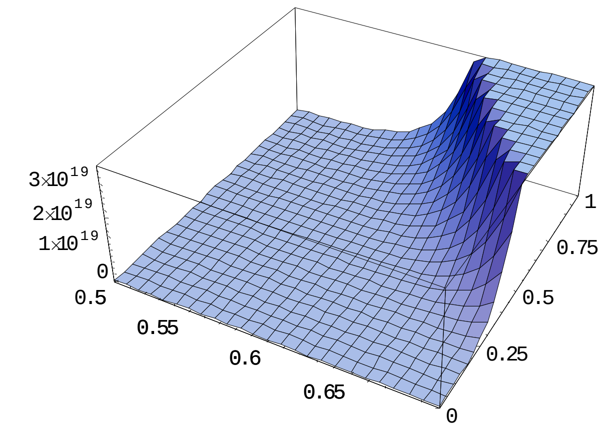
$$\frac{1}{6} a^6 e^{3e^{ax}+ax} \left(1 + 53\, e^{ax} + 318\, e^{2ax} + 515\, e^{3ax} + 279\, e^{4ax} + 45\, e^{5ax}\right) t^3 +$$

$$\frac{1}{24} a^8 e^{4e^{ax}+ax} \left(1 + 236\, e^{ax} + 4304\, e^{2ax} + 20192\, e^{3ax} + 35272\, e^{4ax} + 26312\, e^{5ax} + 8392\, e^{6ax} + 928\, e^{7ax}\right) t^4 +$$

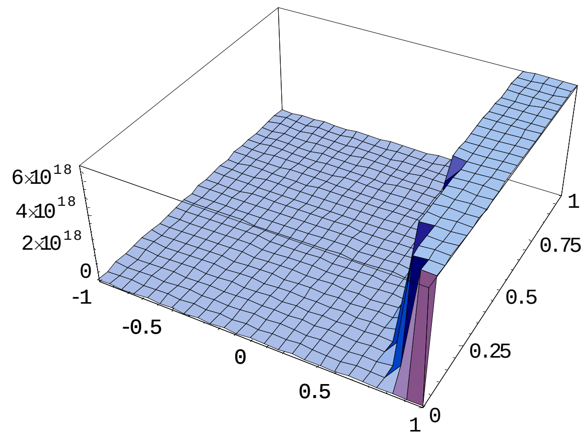
$$\frac{1}{120} a^{10} e^{5e^{ax}+ax} \left(1 + 987\, e^{ax} + 47374\, e^{2ax} + 520713\, e^{3ax} + 2102685\, e^{4ax} + 3836499\, e^{5ax} + 3477200\, e^{6ax} + 1601934\, e^{7ax} + 354385\, e^{8ax} + 29525\, e^{9ax}\right) t^5$$

For $a = 2$ the solution has the behavior

Graph for the range $\{x, 0.5, 0.7\}$, $\{t, 0, 1\}$

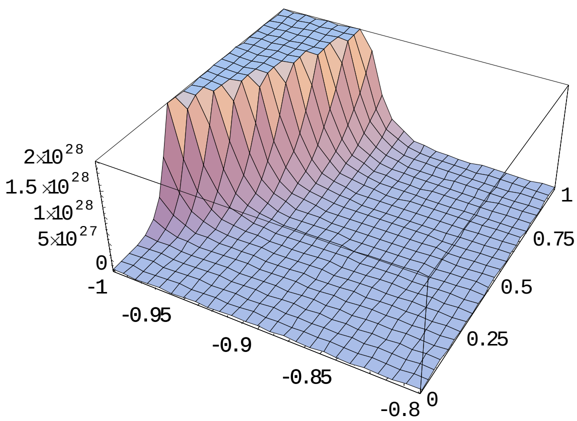


Graph for the range $\{x, -1, 1\}$,$\{t, 0, 1\}$

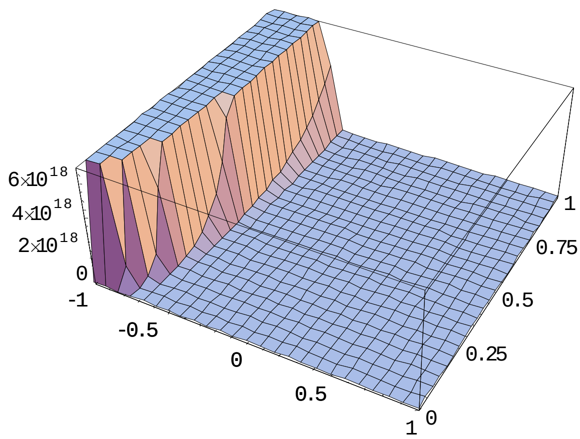


and for $a = -2$ the behavior is as follows.

Graph for the range $\{x, -1, -0.8\}$, $\{t, 0, 1\}$



Graph for the range $\{x, -1, 1\}$,$\{t, 0, 1\}$



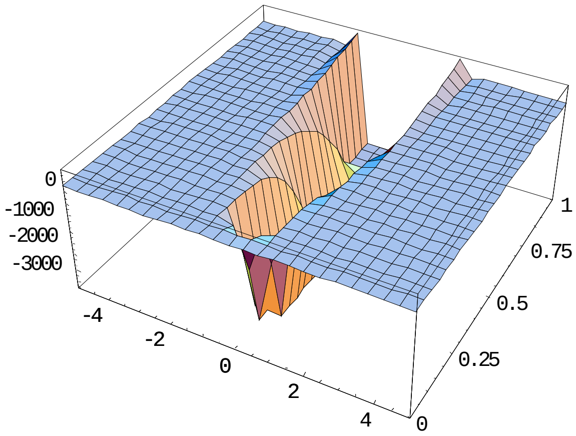
(2-c) $\quad f(u) = e^u, \qquad g(x) = e^{-ax^2}$

The Adomian solution for general '*a*' is given below.

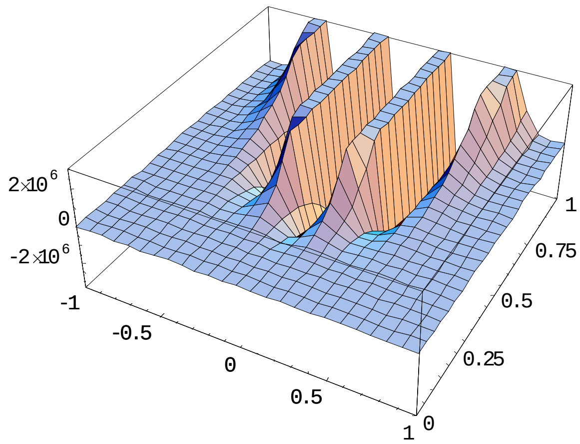
$$
\begin{aligned}
u(x,t) = {} & e^{-a x^2} + 2 a e^{e^{-a x^2} - 2 a x^2} t \left(2 a x^2 + e^{a x^2} (-1 + 2 a x^2)\right) + \\
& 2 a^2 e^{2 e^{-a x^2} - 4 a x^2} t^2 \\
& \quad \Big(16 a^2 x^4 + 28 a e^{a x^2} x^2 (-1 + 2 a x^2) + e^{3 a x^2} (3 - 12 a x^2 + 4 a^2 x^4) + \\
& \qquad e^{2 a x^2} (4 - 52 a x^2 + 40 a^2 x^4)\Big) + \\
& \frac{4}{3} a^3 e^{3 e^{-a x^2} - 6 a x^2} t^3 \Big(360 a^3 x^6 + 1116 a^2 e^{a x^2} x^4 (-1 + 2 a x^2) + \\
& \qquad 10 a e^{2 a x^2} x^2 (69 - 480 a x^2 + 412 a^2 x^4) + \\
& \qquad e^{5 a x^2} (-15 + 90 a x^2 - 60 a^2 x^4 + 8 a^3 x^6) + \\
& \qquad e^{4 a x^2} (-75 + 1098 a x^2 - 1564 a^2 x^4 + 424 a^3 x^6) + \\
& \qquad e^{3 a x^2} (-47 + 1908 a x^2 - 5388 a^2 x^4 + 2544 a^3 x^6)\Big) + \\
& \frac{2}{3} a^4 e^{4 e^{-a x^2} - 8 a x^2} t^4 \Big(14848 a^4 x^8 + 67136 a^3 e^{a x^2} x^6 (-1 + 2 a x^2) + \\
& \qquad 16 a^2 e^{2 a x^2} x^4 (5067 - 29334 a x^2 + 26312 a^2 x^4) + \\
& \qquad 16 a e^{3 a x^2} x^2 (-1629 + 2 a x^2 (13227 + 2 a x^2 (-16723 + 8818 a x^2))) + \\
& \qquad e^{7 a x^2} (105 + 8 a x^2 (-105 + a x^2 (105 + 2 a x^2 (-14 + a x^2)))) + \\
& \qquad 4 e^{6 a x^2} (345 + 4 a x^2 (-1515 + 4 a x^2 (780 + a x^2 (-417 + 59 a x^2)))) + \\
& \qquad 4 e^{5 a x^2} (657 + 2 a x^2 (-11883 + 2 a x^2 (21747 + 2 a x^2 (-9601 + 2152 a x^2)))) + \\
& \qquad 4 e^{4 a x^2} (261 + 4 a x^2 (-6075 + a x^2 (41325 + 4 a x^2 (-14849 + 5048 a x^2))))\Big) + \\
& \frac{4}{15} a^5 e^{5\left(e^{-a x^2} - 2 a x^2\right)} t^5 \\
& \quad \Big(944800 a^5 x^{10} + 5670160 a^4 e^{a x^2} x^8 (-1 + 2 a x^2) + \\
& \qquad 16 a^3 e^{2 a x^2} x^6 (659143 + 4 a x^2 (-871879 + 800967 a x^2)) + \\
& \qquad 200 a^2 e^{3 a x^2} x^4 (-34935 + 4 a x^2 (104316 + a x^2 (-249547 + 139088 a x^2))) + \\
& \qquad 2 a e^{4 a x^2} x^2 (693465 + 4 a x^2 (-5428185 + 4 a x^2 (7167820 + a x^2 (-10212925 + 3836499 a x^2)))) + \\
& \qquad e^{9 a x^2} (-945 + 2 a x^2 (4725 + 4 a x^2 (-1575 + 2 a x^2 (315 + a x^2 (-45 + 2 a x^2))))) + \\
& \qquad e^{8 a x^2} (-28395 + 2 a x^2 (296775 + 4 a x^2 (-199965 + 2 a x^2 (79649 + a x^2 (-22511 + 1974 a x^2))))) + \\
& \qquad 2 e^{6 a x^2} (-64179 + a x^2 (4675773 + 8 a x^2 (-4368687 + a x^2 (8528899 + 18 a x^2 (-304519 + 57857 a x^2))))) + \\
& \qquad e^{7 a x^2} (-117933 + 4 a x^2 (1122855 + 2 a x^2 (-2534883 + 2 a x^2 (1613694 + a x^2 (-708377 + 94748 a x^2))))) + \\
& \qquad e^{5 a x^2} (-36669 + 2 a x^2 (3290127 + 8 a x^2 (-5510685 + a x^2 (16888936 + 5 a x^2 (-3170287 + 841074 a x^2)))))\Big)
\end{aligned}
$$

Some properties of the solution for $a = 2$ are shown by the following graphs.

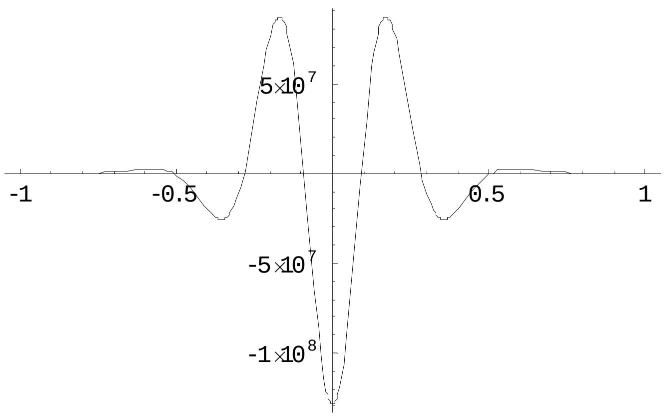
Graph for the range {*x*, -5,5}, {*t*, 0, 1}



Graph for the range {*x*, -1, 1},{*t*, 0, 1}



For the fixed time $t = 0.8$ the behavior of the solution is



(2-d) $f(u)=e^{u}$, $g(x)=\operatorname{sech}^2 x$

The Adomian solution is

$$u(x,t)=\text{Sech}[x]^2+\frac{1}{2}e^{\text{Sech}[x]^2}\,t\,(-7+2\,\text{Cosh}[2x]+\text{Cosh}[4x])\,\text{Sech}[x]^6+$$

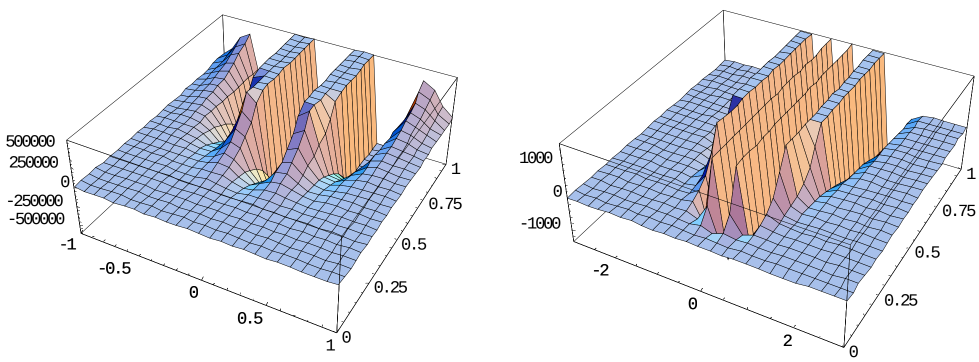
$$\frac{1}{64}e^{2\,\text{Sech}[x]^2}t^2\,(2764-406\,\text{Cosh}[2x]-1248\,\text{Cosh}[4x]-107\,\text{Cosh}[6x]+20\,\text{Cosh}[8x]+\text{Cosh}[10x])\,\text{Sech}[x]^{12}+$$

$$\frac{1}{3072}\Big(e^{3\,\text{Sech}[x]^2}t^3\,(-3365157+19202\,\text{Cosh}[2x]+2400116\,\text{Cosh}[4x]+377814\,\text{Cosh}[6x]-142556\,\text{Cosh}[8x]-30526\,\text{Cosh}[10x]-372\,\text{Cosh}[12x]+102\,\text{Cosh}[14x]+\text{Cosh}[16x])\,\text{Sech}[x]^{18}\Big)+$$

$$\frac{1}{196608}\Big(e^{4\,\text{Sech}[x]^2}t^4\,(8185299120+1001217750\,\text{Cosh}[2x]-7496138224\,\text{Cosh}[4x]-1733612582\,\text{Cosh}[6x]+800297664\,\text{Cosh}[8x]+284163125\,\text{Cosh}[10x]+2415656\,\text{Cosh}[12x]-6588701\,\text{Cosh}[14x]-537456\,\text{Cosh}[16x]+567\,\text{Cosh}[18x]+456\,\text{Cosh}[20x]+\text{Cosh}[22x])\,\text{Sech}[x]^{24}\Big)+$$

$$\frac{1}{15728640}\Big(e^{5\,\text{Sech}[x]^2}t^5\,(-32987412576380-7480572111768\,\text{Cosh}[2x]+35069821348827\,\text{Cosh}[4x]+10736873036454\,\text{Cosh}[6x]-5235938998102\,\text{Cosh}[8x]-2583899128290\,\text{Cosh}[10x]-17819499591\,\text{Cosh}[12x]+138229868956\,\text{Cosh}[14x]+18870732444\,\text{Cosh}[16x]-537206468\,\text{Cosh}[18x]-213675925\,\text{Cosh}[20x]-8683406\,\text{Cosh}[22x]+26934\,\text{Cosh}[24x]+1930\,\text{Cosh}[26x]+\text{Cosh}[28x])\,\text{Sech}[x]^{30}\Big)$$

The variations in the solution are shown by the graphs below.



3.3 Case 3

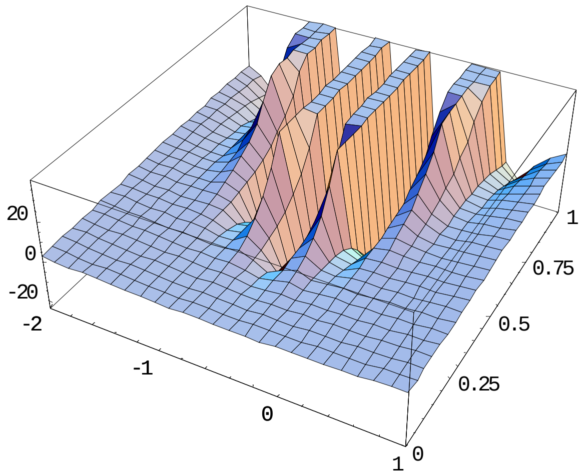
We consider the nonlinear heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(f(u)\frac{\partial u}{\partial x} \right), \qquad f(u) = \ln u$$
$$u(x,0) = g(x)$$

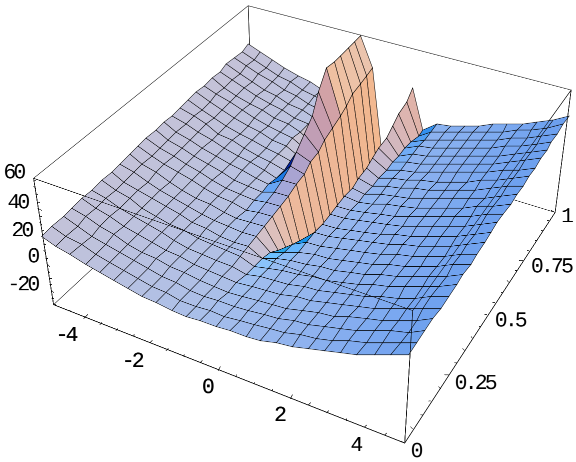
(3-a) $f(u) = \ln u$, $g(x) = ax^2 + bx + c$

The Adomian solution $u(x,t)$ for general a, b, c and m can be obtained from authors as Mathematica file. For the case $a = b = c = 1$ the graphs of the solution are

Graph for the range {x, -2,1}, {t, 0, 1}



Graph for the range {x, -5, 5},{t, 0, 1}



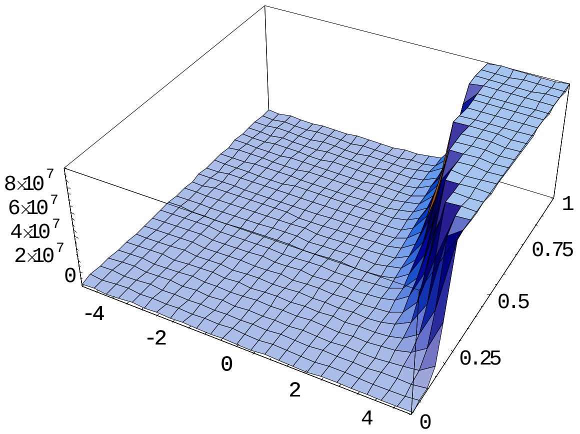
(3-b) $f(u) = \ln u$, $g(x) = e^{ax}$

The Adomian solution for general 'a' is given by

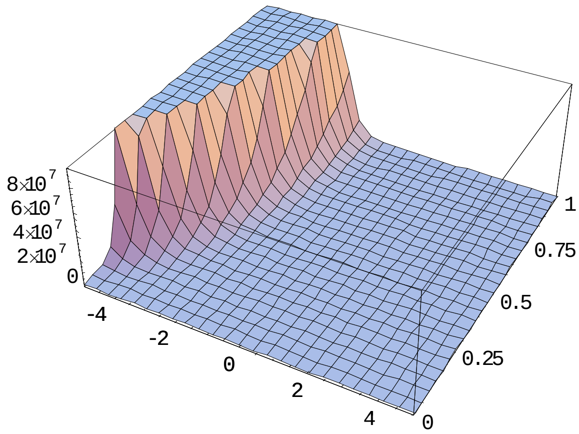
$$u(x, t) = e^{ax} + a^2 e^{ax} t\,(1 + \mathrm{Log}[e^{ax}]) + \frac{1}{2} a^4 e^{ax} t^2 \left(4 + 5\,\mathrm{Log}[e^{ax}] + \mathrm{Log}[e^{ax}]^2\right) +$$
$$\frac{1}{6} a^6 e^{ax} t^3 (1 + \mathrm{Log}[e^{ax}])\,(25 + \mathrm{Log}[e^{ax}]\,(11 + \mathrm{Log}[e^{ax}])) +$$
$$\frac{1}{24} a^8 e^{ax} t^4 (1 + \mathrm{Log}[e^{ax}])\,(211 + \mathrm{Log}[e^{ax}]\,(126 + \mathrm{Log}[e^{ax}]\,(21 + \mathrm{Log}[e^{ax}]))) +$$
$$\frac{1}{120} a^{10} e^{ax} t^5 (1 + \mathrm{Log}[e^{ax}])\,(2236 +$$
$$\mathrm{Log}[e^{ax}]\,(1639 + \mathrm{Log}[e^{ax}]\,(381 + \mathrm{Log}[e^{ax}]\,(34 + \mathrm{Log}[e^{ax}]))))$$

For $a = 2$ and $a = -2$, the behavior of the solution is

Graph for $a = 2$ for the range {x, -5,5}, {t, 0, 1}



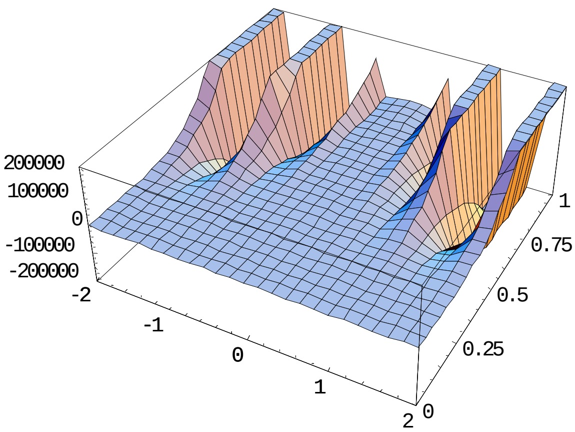
Graph for $a = -2$ for the range {x, -5,5}, {t, 0, 1}



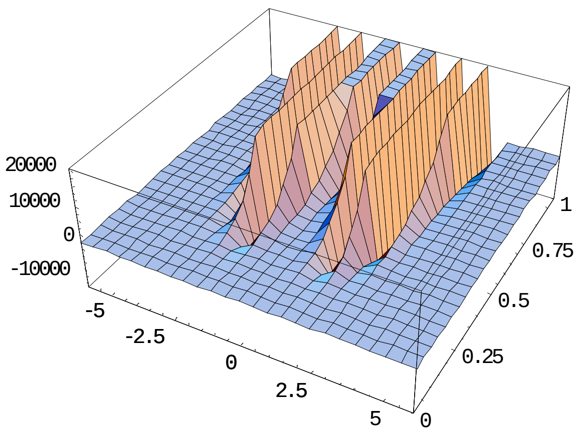
(3-c) $f(u) = \ln u$, $g(x) = e^{-ax^2}$

Solution for general 'a' can be obtained from authors as Mathematica file. The nature of the solution for $a = 2$ is shown by the following graphs.

Graph for the range {x, -2,2}, {t, 0, 1}



Graph for the range {x, -6,6}, {t, 0, 1}



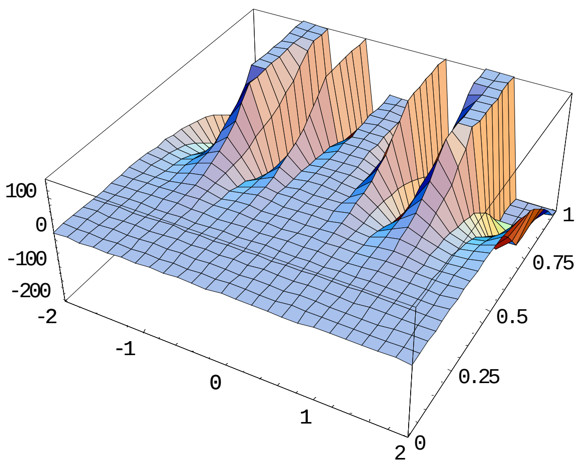
(3-d) $f(u) = \ln u\,,\quad g(x) = \mathrm{sech}^2 x$

The Adomian solution is

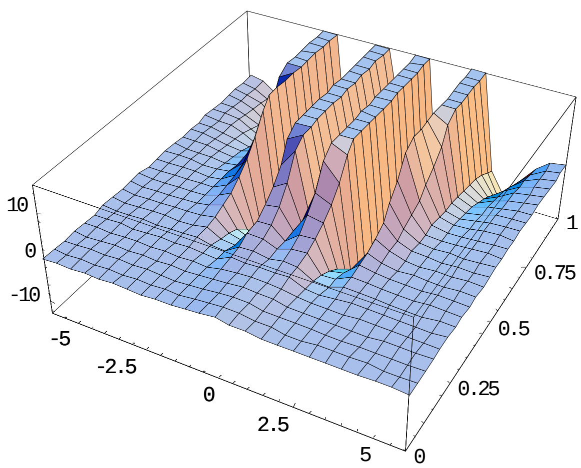
$$
\begin{aligned}
u(x,t) =\;& \mathrm{Sech}[x]^2 + 2\,t\left(-1 + \mathrm{Cosh}[2x] + (-2 + \mathrm{Cosh}[2x])\,\mathrm{Log}\left[\mathrm{Sech}[x]^2\right]\right)\mathrm{Sech}[x]^4 + \\
& \frac{1}{3}t^3 \\
& \Big(-1460 + 1899\,\mathrm{Cosh}[2x] - 464\,\mathrm{Cosh}[4x] + 25\,\mathrm{Cosh}[6x] + \\
& \quad 4\,(-1408 + 1677\,\mathrm{Cosh}[2x] - 306\,\mathrm{Cosh}[4x] + 9\,\mathrm{Cosh}[6x])\,\mathrm{Log}\left[\mathrm{Sech}[x]^2\right] + \\
& \quad 6\,(-871 + 942\,\mathrm{Cosh}[2x] - 127\,\mathrm{Cosh}[4x] + 2\,\mathrm{Cosh}[6x])\,\mathrm{Log}\left[\mathrm{Sech}[x]^2\right]^2 + \\
& \quad (-1208 + 1191\,\mathrm{Cosh}[2x] - 120\,\mathrm{Cosh}[4x] + \mathrm{Cosh}[6x])\,\mathrm{Log}\left[\mathrm{Sech}[x]^2\right]^3\Big)\,\mathrm{Sech}[x]^8 + \\
& \frac{1}{12}t^4\Big(138561 - 195232\,\mathrm{Cosh}[2x] + 64588\,\mathrm{Cosh}[4x] - 8128\,\mathrm{Cosh}[6x] + 211\,\mathrm{Cosh}[8x] + \\
& \quad (697847 - 932862\,\mathrm{Cosh}[2x] + 260696\,\mathrm{Cosh}[4x] - 23778\,\mathrm{Cosh}[6x] + \\
& \qquad 337\,\mathrm{Cosh}[8x])\,\mathrm{Log}\left[\mathrm{Sech}[x]^2\right] + (1015093 - 1281554\,\mathrm{Cosh}[2x] + \\
& \qquad 299496\,\mathrm{Cosh}[4x] - 19614\,\mathrm{Cosh}[6x] + 147\,\mathrm{Cosh}[8x])\,\mathrm{Log}\left[\mathrm{Sech}[x]^2\right]^2 + \\
& \quad 2\,(256853 - 306194\,\mathrm{Cosh}[2x] + 59936\,\mathrm{Cosh}[4x] - 2830\,\mathrm{Cosh}[6x] + \\
& \qquad 11\,\mathrm{Cosh}[8x])\,\mathrm{Log}\left[\mathrm{Sech}[x]^2\right]^3 + (78095 - 88234\,\mathrm{Cosh}[2x] + \\
& \qquad 14608\,\mathrm{Cosh}[4x] - 502\,\mathrm{Cosh}[6x] + \mathrm{Cosh}[8x])\,\mathrm{Log}\left[\mathrm{Sech}[x]^2\right]^4\Big)\,\mathrm{Sech}[x]^{10} + \\
& t^2\,\mathrm{Sech}[x]^6 \\
& \Big((73 - 70\,\mathrm{Cosh}[2x] + 5\,\mathrm{Cosh}[4x])\,\mathrm{Log}\left[\mathrm{Sech}[x]^2\right] + \\
& \quad (33 - 26\,\mathrm{Cosh}[2x] + \mathrm{Cosh}[4x])\,\mathrm{Log}\left[\mathrm{Sech}[x]^2\right]^2 + 4\,(-11 + 4\,\mathrm{Cosh}[2x])\,\mathrm{Sinh}[x]^2\Big) + \\
& \frac{1}{60}t^5\,\mathrm{Sech}[x]^{12} \\
& \Big((176449582\,\mathrm{Cosh}[2x] - 63293104\,\mathrm{Cosh}[4x] + 10001199\,\mathrm{Cosh}[6x] - \\
& \qquad 324\,(378703 + 1605\,\mathrm{Cosh}[8x]) + 3875\,\mathrm{Cosh}[10x])\,\mathrm{Log}\left[\mathrm{Sech}[x]^2\right] + \\
& \quad 2\,(-124972845 + 173031932\,\mathrm{Cosh}[2x] - 55174280\,\mathrm{Cosh}[4x] + \\
& \qquad 7057314\,\mathrm{Cosh}[6x] - 260315\,\mathrm{Cosh}[8x] + 1010\,\mathrm{Cosh}[10x])\,\mathrm{Log}\left[\mathrm{Sech}[x]^2\right]^2 + \\
& \quad (-205978988 + 274356206\,\mathrm{Cosh}[2x] - 77664096\,\mathrm{Cosh}[4x] + \\
& \qquad 8042227\,\mathrm{Cosh}[6x] - 210420\,\mathrm{Cosh}[8x] + 415\,\mathrm{Cosh}[10x])\,\mathrm{Log}\left[\mathrm{Sech}[x]^2\right]^3 + \\
& \quad (-69677278 + 89377062\,\mathrm{Cosh}[2x] - 22539312\,\mathrm{Cosh}[4x] + 1900343\,\mathrm{Cosh}[6x] - \\
& \qquad 35410\,\mathrm{Cosh}[8x] + 35\,\mathrm{Cosh}[10x])\,\mathrm{Log}\left[\mathrm{Sech}[x]^2\right]^4 + \\
& \quad (-7862124 + 9738114\,\mathrm{Cosh}[2x] - 2203488\,\mathrm{Cosh}[4x] + \\
& \qquad 152637\,\mathrm{Cosh}[6x] - 2036\,\mathrm{Cosh}[8x] + \mathrm{Cosh}[10x])\,\mathrm{Log}\left[\mathrm{Sech}[x]^2\right]^5 + \\
& \quad 8\,(2953334 - 3784339\,\mathrm{Cosh}[2x] + 959912\,\mathrm{Cosh}[4x] - \\
& \qquad 78985\,\mathrm{Cosh}[6x] + 1118\,\mathrm{Cosh}[8x])\,\mathrm{Sinh}[x]^2\Big)
\end{aligned}
$$

An idea about the behavior of the solution can be obtained by the following graphs.

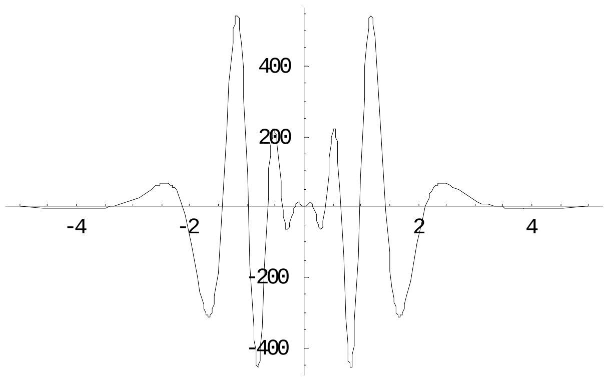
Graph for the range {x, -2,2}, {t, 0, 1}



Graph for the range {x, -6,6}, {t, 0, 1}



For the fixed time $t = 0.8$ the behavior of the solution is



3.4 Case 4

We consider the nonlinear heat equation

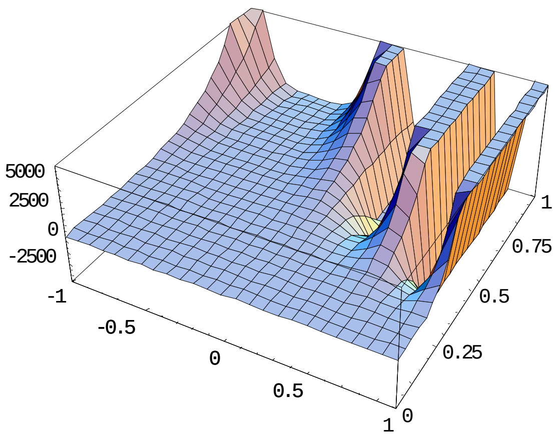
$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(f(u)\frac{\partial u}{\partial x}\right), \qquad f(u) = \sin u$$

$$u(x,0) = g(x)$$

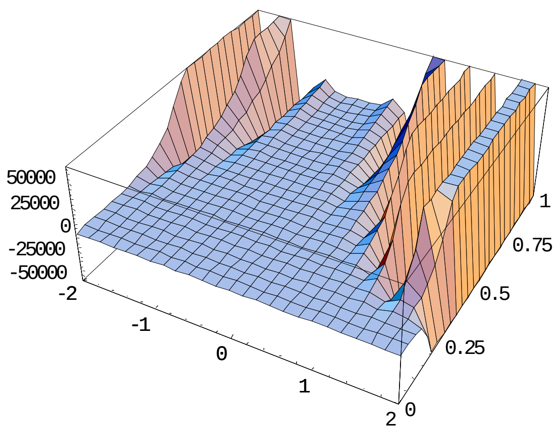
(4-a) $f(u) = \sin u$, $g(x) = x^2 + x + 1$

The Adomian solution $u(x,t)$ can be obtained from authors as Mathematica file. The nature of solution can be seen from the following graphs.

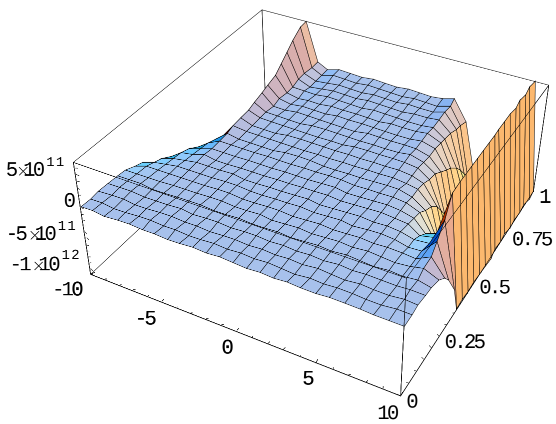
Graph for the range {*x*, -1,1}, {*t*, 0, 1}



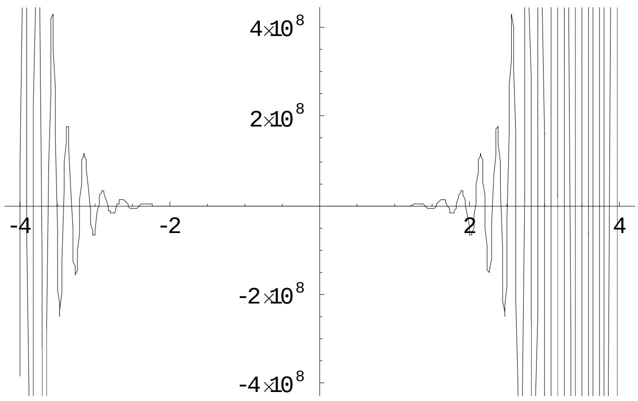
Graph for the range {*x*, -2,2}, {*t*, 0, 1}



Graph for the range {*x*, -10,10}, {*t*, 0, 1}



Graph for fixed t=0.8 for the range {*x*, -4,4}



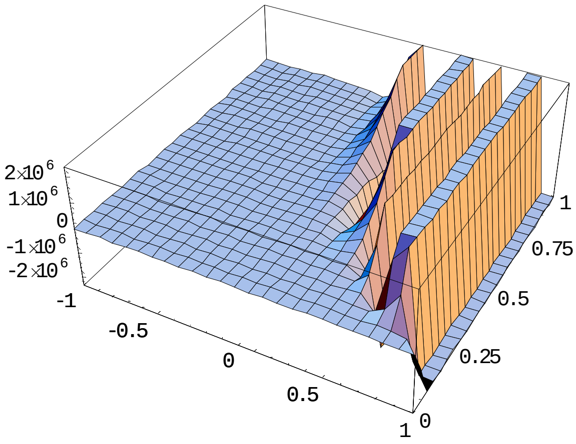
(4-b) $f(u) = \sin u\,,\quad g(x) = e^{ax}$

The Adomian solution for general *'a'* is

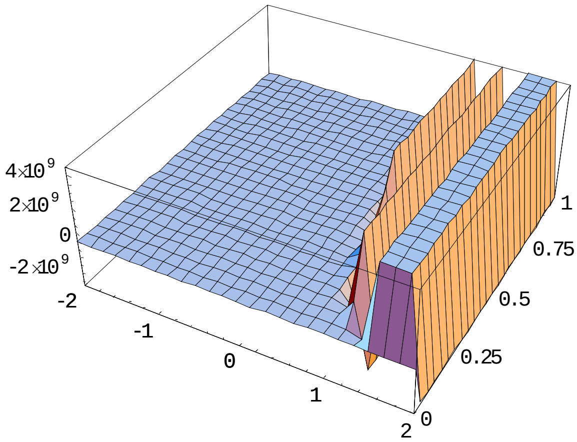
$$
\begin{aligned}
u(x,t) = {} & e^{ax} + a^2 e^{ax}\, t\, (e^{ax} \mathrm{Cos}[e^{ax}] + \mathrm{Sin}[e^{ax}]) + \\
& \frac{1}{24} a^6 e^{ax} t^3 \\
& \Big(e^{ax} \left(53 - 111 e^{2ax} + e^{4ax}\right) \mathrm{Cos}[e^{ax}] + 4 \mathrm{Sin}[e^{ax}]^3 + \\
& \quad e^{ax} \\
& \quad \Big(\left(-53 + 515 e^{2ax} - 45 e^{4ax}\right) \mathrm{Cos}[3 e^{ax}] + \\
& \qquad 2 e^{ax} \left(76 - 129 e^{2ax} + \left(318 - 279 e^{2ax}\right) \mathrm{Cos}[2 e^{ax}]\right) \mathrm{Sin}[e^{ax}]\Big)\Big) - \\
& \frac{1}{4} a^4 e^{ax} t^2 \left(-1 + \left(1 - 14 e^{2ax}\right) \mathrm{Cos}[2 e^{ax}] + 2 e^{ax} \left(-5 + 2 e^{2ax}\right) \mathrm{Sin}[2 e^{ax}]\right) + \\
& \frac{1}{192} a^8 e^{ax} t^4 \Big(3 - 1224 e^{2ax} + 1200 e^{4ax} + \\
& \quad 4 \left(-1 + 1382 e^{2ax} - 4412 e^{4ax} + 346 e^{6ax}\right) \mathrm{Cos}[2 e^{ax}] + \\
& \quad \left(1 - 4304 e^{2ax} + 35272 e^{4ax} - 8392 e^{6ax}\right) \mathrm{Cos}[4 e^{ax}] + 472 e^{ax} \mathrm{Sin}[2 e^{ax}] - \\
& \quad 16432 e^{3ax} \mathrm{Sin}[2 e^{ax}] + 7760 e^{5ax} \mathrm{Sin}[2 e^{ax}] - 80 e^{7ax} \mathrm{Sin}[2 e^{ax}] - \\
& \quad 236 e^{ax} \mathrm{Sin}[4 e^{ax}] + 20192 e^{3ax} \mathrm{Sin}[4 e^{ax}] - 26312 e^{5ax} \mathrm{Sin}[4 e^{ax}] + \\
& \quad 928 e^{7ax} \mathrm{Sin}[4 e^{ax}]\Big) + \\
& \frac{1}{1920} \\
& \Big(a^{10} e^{ax} t^5 \Big(2 e^{ax} \left(987 - 146005 e^{2ax} + 252687 e^{4ax} - 23762 e^{6ax} + 57 e^{8ax}\right) \mathrm{Cos}[e^{ax}] - \\
& \quad e^{ax} \left(2961 - 812723 e^{2ax} + 3092097 e^{4ax} - 631394 e^{6ax} + 5175 e^{8ax}\right) \mathrm{Cos}[3 e^{ax}] + \\
& \quad 987 e^{ax} \mathrm{Cos}[5 e^{ax}] - 520713 e^{3ax} \mathrm{Cos}[5 e^{ax}] + 3836499 e^{5ax} \mathrm{Cos}[5 e^{ax}] - \\
& \quad 1601934 e^{7ax} \mathrm{Cos}[5 e^{ax}] + 29525 e^{9ax} \mathrm{Cos}[5 e^{ax}] + 10 \mathrm{Sin}[e^{ax}] - \\
& \quad 69796 e^{2ax} \mathrm{Sin}[e^{ax}] + 596498 e^{4ax} \mathrm{Sin}[e^{ax}] - 228120 e^{6ax} \mathrm{Sin}[e^{ax}] + \\
& \quad 4082 e^{8ax} \mathrm{Sin}[e^{ax}] - 5 \mathrm{Sin}[3 e^{ax}] + 102222 e^{2ax} \mathrm{Sin}[3 e^{ax}] - \\
& \quad 2368625 e^{4ax} \mathrm{Sin}[3 e^{ax}] + 1977816 e^{6ax} \mathrm{Sin}[3 e^{ax}] - \\
& \quad 94557 e^{8ax} \mathrm{Sin}[3 e^{ax}] + \mathrm{Sin}[5 e^{ax}] - 47374 e^{2ax} \mathrm{Sin}[5 e^{ax}] + \\
& \quad 2102685 e^{4ax} \mathrm{Sin}[5 e^{ax}] - 3477200 e^{6ax} \mathrm{Sin}[5 e^{ax}] + 354385 e^{8ax} \mathrm{Sin}[5 e^{ax}]\Big)\Big)
\end{aligned}
$$

Some graphs of the function for $a = 2$ are given below.

Graph for the range {x, -1,1}, {t, 0, 1}



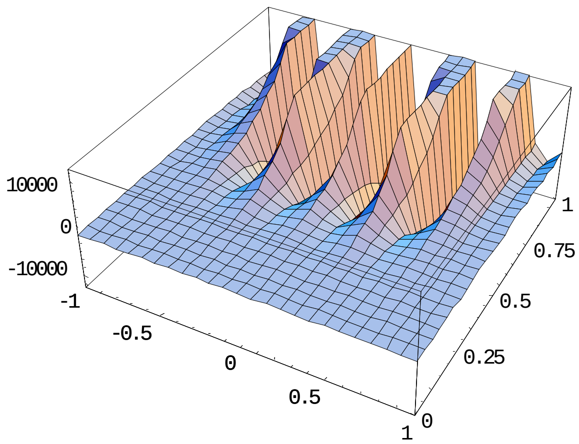
Graph for the range {x, -2,2}, {t, 0, 1}



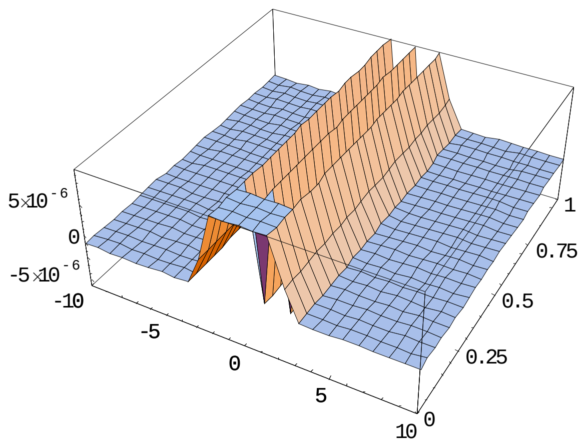
(4-c) $f(u) = \sin u$, $g(x) = e^{-ax^2}$

Solution for general 'a' can be obtained from authors as Mathematica file. The nature of the solution for $a = 2$ is shown by the following graphs.

Graph for the range {x, -1,1}, {t, 0, 1}



Graph for the range {x, -10,10}, {t, 0, 1}



(4-d) $f(u)=\sin u\,,\quad g(x)=\operatorname{sech}^2 x$

The Adomian solution is

$$u(x,t) = \mathrm{Sech}[x]^2 - \frac{1}{12288}\Big(t^3\,\mathrm{Sech}[x]^{18}\Big(248688\,\mathrm{Cos}[\mathrm{Sech}[x]^2] + 7824\,\mathrm{Cos}[3\,\mathrm{Sech}[x]^2] + 219746\,\mathrm{Cosh}[2x - i\,\mathrm{Sech}[x]^2] +$$

$$50244\,\mathrm{Cosh}[4x - i\,\mathrm{Sech}[x]^2] -$$

$$133042\,\mathrm{Cosh}[6x - i\,\mathrm{Sech}[x]^2] - 66744\,\mathrm{Cosh}[8x - i\,\mathrm{Sech}[x]^2] + 7610\,\mathrm{Cosh}[10x - i\,\mathrm{Sech}[x]^2] +$$

$$2748\,\mathrm{Cosh}[12x - i\,\mathrm{Sech}[x]^2] - 106\,\mathrm{Cosh}[14x - i\,\mathrm{Sech}[x]^2] + 219746\,\mathrm{Cosh}[2x + i\,\mathrm{Sech}[x]^2] +$$

$$50244\,\mathrm{Cosh}[4x + i\,\mathrm{Sech}[x]^2] - 133042\,\mathrm{Cosh}[6x + i\,\mathrm{Sech}[x]^2] - 66744\,\mathrm{Cosh}[8x + i\,\mathrm{Sech}[x]^2] +$$

$$7610\,\mathrm{Cosh}[10x + i\,\mathrm{Sech}[x]^2] + 2748\,\mathrm{Cosh}[12x + i\,\mathrm{Sech}[x]^2] - 106\,\mathrm{Cosh}[14x + i\,\mathrm{Sech}[x]^2] +$$

$$126622\,\mathrm{Cosh}[2x - 3i\,\mathrm{Sech}[x]^2] - 633924\,\mathrm{Cosh}[4x - 3i\,\mathrm{Sech}[x]^2] + 160050\,\mathrm{Cosh}[6x - 3i\,\mathrm{Sech}[x]^2] +$$

$$161720\,\mathrm{Cosh}[8x - 3i\,\mathrm{Sech}[x]^2] - 20538\,\mathrm{Cosh}[10x - 3i\,\mathrm{Sech}[x]^2] - 2748\,\mathrm{Cosh}[12x - 3i\,\mathrm{Sech}[x]^2] +$$

$$106\,\mathrm{Cosh}[14x - 3i\,\mathrm{Sech}[x]^2] + 126622\,\mathrm{Cosh}[2x + 3i\,\mathrm{Sech}[x]^2] - 633924\,\mathrm{Cosh}[4x + 3i\,\mathrm{Sech}[x]^2] +$$

$$160050\,\mathrm{Cosh}[6x + 3i\,\mathrm{Sech}[x]^2] + 161720\,\mathrm{Cosh}[8x + 3i\,\mathrm{Sech}[x]^2] - 20538\,\mathrm{Cosh}[10x + 3i\,\mathrm{Sech}[x]^2] -$$

$$2748\,\mathrm{Cosh}[12x + 3i\,\mathrm{Sech}[x]^2] + 106\,\mathrm{Cosh}[14x + 3i\,\mathrm{Sech}[x]^2] - 191841\,\mathrm{Sin}[\mathrm{Sech}[x]^2] +$$

$$149067\,\mathrm{Sin}[3\,\mathrm{Sech}[x]^2] - 12969\,i\,\mathrm{Sinh}[2x - i\,\mathrm{Sech}[x]^2] + 214278\,i\,\mathrm{Sinh}[4x - i\,\mathrm{Sech}[x]^2] +$$

$$108821\,i\,\mathrm{Sinh}[6x - i\,\mathrm{Sech}[x]^2] - 44878\,i\,\mathrm{Sinh}[8x - i\,\mathrm{Sech}[x]^2] - 21265\,i\,\mathrm{Sinh}[10x - i\,\mathrm{Sech}[x]^2] +$$

$$1274\,i\,\mathrm{Sinh}[12x - i\,\mathrm{Sech}[x]^2] + 165\,i\,\mathrm{Sinh}[14x - i\,\mathrm{Sech}[x]^2] - \frac{3}{2}\,i\,\mathrm{Sinh}[16x - i\,\mathrm{Sech}[x]^2] +$$

$$12969\,i\,\mathrm{Sinh}[2x + i\,\mathrm{Sech}[x]^2] - 214278\,i\,\mathrm{Sinh}[4x + i\,\mathrm{Sech}[x]^2] - 108821\,i\,\mathrm{Sinh}[6x + i\,\mathrm{Sech}[x]^2] +$$

$$44878\,i\,\mathrm{Sinh}[8x + i\,\mathrm{Sech}[x]^2] + 21265\,i\,\mathrm{Sinh}[10x + i\,\mathrm{Sech}[x]^2] - 1274\,i\,\mathrm{Sinh}[12x + i\,\mathrm{Sech}[x]^2] -$$

$$165\,i\,\mathrm{Sinh}[14x + i\,\mathrm{Sech}[x]^2] + \frac{3}{2}\,i\,\mathrm{Sinh}[16x + i\,\mathrm{Sech}[x]^2] + 428387\,i\,\mathrm{Sinh}[2x - 3i\,\mathrm{Sech}[x]^2] -$$

$$138274\,i\,\mathrm{Sinh}[4x - 3i\,\mathrm{Sech}[x]^2] - 446919\,i\,\mathrm{Sinh}[6x - 3i\,\mathrm{Sech}[x]^2] + 79578\,i\,\mathrm{Sinh}[8x - 3i\,\mathrm{Sech}[x]^2] +$$

$$31899\,i\,\mathrm{Sinh}[10x - 3i\,\mathrm{Sech}[x]^2] - 2526\,i\,\mathrm{Sinh}[12x - 3i\,\mathrm{Sech}[x]^2] - 55\,i\,\mathrm{Sinh}[14x - 3i\,\mathrm{Sech}[x]^2] +$$

$$\frac{1}{2}\,i\,\mathrm{Sinh}[16x - 3i\,\mathrm{Sech}[x]^2] - 428387\,i\,\mathrm{Sinh}[2x + 3i\,\mathrm{Sech}[x]^2] + 138274\,i\,\mathrm{Sinh}[4x + 3i\,\mathrm{Sech}[x]^2] +$$

$$446919\,i\,\mathrm{Sinh}[6x + 3i\,\mathrm{Sech}[x]^2] - 79578\,i\,\mathrm{Sinh}[8x + 3i\,\mathrm{Sech}[x]^2] - 31899\,i\,\mathrm{Sinh}[10x + 3i\,\mathrm{Sech}[x]^2] +$$

$$2526\,i\,\mathrm{Sinh}[12x + 3i\,\mathrm{Sech}[x]^2] + 55\,i\,\mathrm{Sinh}[14x + 3i\,\mathrm{Sech}[x]^2] - \frac{1}{2}\,i\,\mathrm{Sinh}[16x + 3i\,\mathrm{Sech}[x]^2]\Big)\Big) +$$

$$2t\,\mathrm{Sech}[x]^4\left((-2 + \mathrm{Cosh}[2x])\,\mathrm{Sin}[\mathrm{Sech}[x]^2] + 2\,\mathrm{Cos}[\mathrm{Sech}[x]^2]\,\mathrm{Tanh}[x]^2\right) -$$

$$2t^2\,\mathrm{Sech}[x]^2$$

$$\left(2\,\mathrm{Sech}[x]^2\left(1 + 9\,\mathrm{Tanh}[x]^2\right)\left(\mathrm{Sin}[\mathrm{Sech}[x]^2]^2 - \mathrm{Sin}[2\,\mathrm{Sech}[x]^2]\,\mathrm{Tanh}[x]^2\right) -\right.$$

$$2\,\mathrm{Sin}[\mathrm{Sech}[x]^2]^2\left(1 - 3\,\mathrm{Tanh}[x]^2 + 4\,\mathrm{Tanh}[x]^4\right) +$$

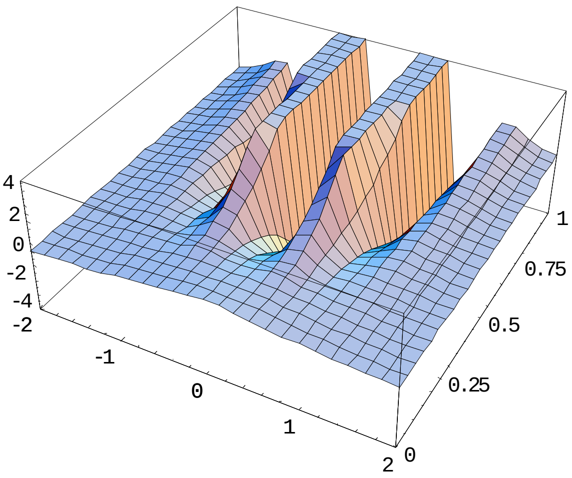
$$\mathrm{Sech}[x]^4\left(-4\,\mathrm{Sin}[\mathrm{Sech}[x]^2]^2 + \mathrm{Sin}[2\,\mathrm{Sech}[x]^2] + \left(-4\,\mathrm{Cos}[2\,\mathrm{Sech}[x]^2] + 33\,\mathrm{Sin}[2\,\mathrm{Sech}[x]^2]\right)\mathrm{Tanh}[x]^2 -\right.$$

$$\left.24\,\mathrm{Cos}[2\,\mathrm{Sech}[x]^2]\,\mathrm{Tanh}[x]^4\right) + \mathrm{Sech}[x]^6\left(-3\,\mathrm{Sin}[2\,\mathrm{Sech}[x]^2] +\right.$$

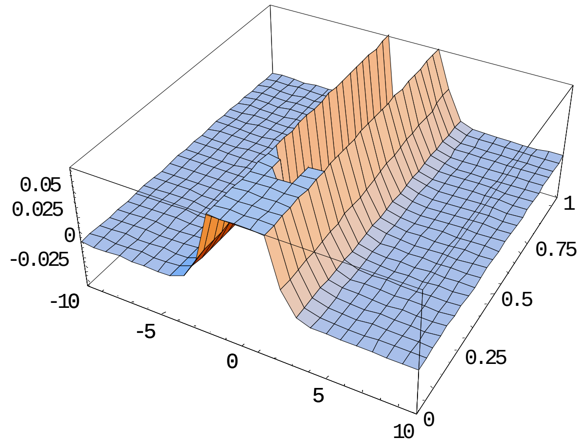
$$\left.\left.18\,\mathrm{Cos}[2\,\mathrm{Sech}[x]^2]\,\mathrm{Tanh}[x]^2 + 8\,\mathrm{Sin}[2\,\mathrm{Sech}[x]^2]\,\mathrm{Tanh}[x]^4\right)\right)$$

The behavior of the solution can be seen from the following graphs.

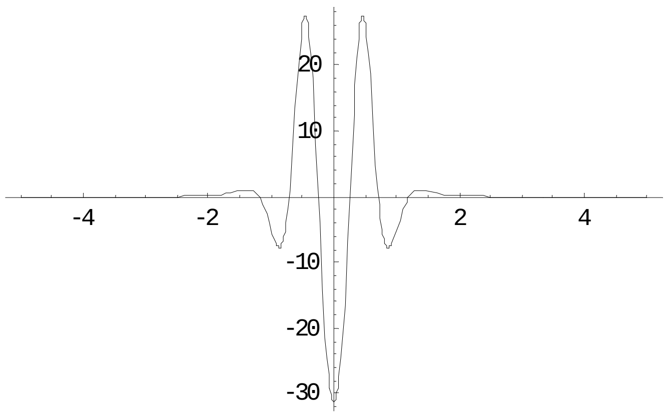
Graph for the range $\{x, -2,2\}$, $\{t, 0, 1\}$



Graph for the range $\{x, -10,10\}$, $\{t, 0, 1\}$



Graph for fixed t=0.8 for the range $\{x, -5,5\}$



3.5 Case 5

We consider the nonlinear heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(f(u)\frac{\partial u}{\partial x}\right), \qquad f(u) = e^{-u}$$

$$u(x,0) = g(x)$$

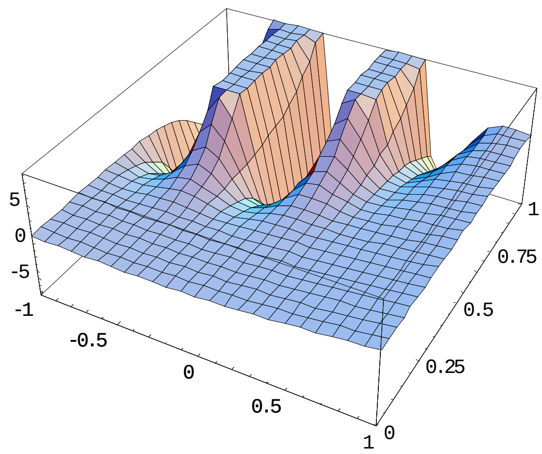
(5-a) $f(u) = e^{-u}$, $g(x) = ax^2 + bx + c$

The Adomian solution $u(x,t)$ for general a, b, c is found as

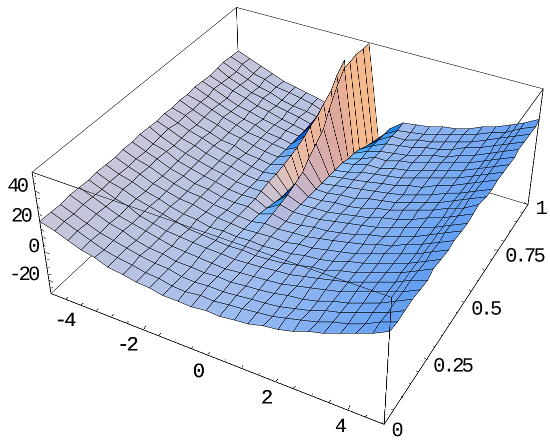
$$\begin{aligned}
u(x,t) = {} & c + b\,x + a\,x^2 - 2\,e^{-2\,(c+x\,(b+a\,x))}\,t^2\,\big(b^4 + 16\,a^4\,x^4 + \\
& 4\,a^2\,(-1 + b\,x)\,(-1 + 6\,b\,x) + a\,b^2\,(-7 + 8\,b\,x) + 4\,a^3\,x^2\,(-7 + 8\,b\,x)\big) - \\
& e^{-c-x\,(b+a\,x)}\,t\,\big(b^2 + 2\,a\,(-1 + 2\,x\,(b + a\,x))\big) - \\
& \frac{1}{6}\,e^{-3\,(c+x\,(b+a\,x))}\,t^3\,\big(45\,b^6 + 2880\,a^6\,x^6 + 18\,a\,b^4\,(-31 + 30\,b\,x) + \\
& 288\,a^5\,x^4\,(-31 + 30\,b\,x) + 12\,a^2\,b^2\,(115 + 3\,b\,x\,(-124 + 75\,b\,x)) + \\
& 48\,a^4\,x^2\,(115 + 3\,b\,x\,(-124 + 75\,b\,x)) + \\
& 8\,a^3\,(-47 + 6\,b\,x\,(115 + 3\,b\,x\,(-93 + 50\,b\,x)))\big) - \\
& \frac{2}{3}\,e^{-4\,(c+x\,(b+a\,x))}\,t^4\,\big(58\,b^8 + 14848\,a^8\,x^8 + a\,b^6\,(-1049 + 928\,b\,x) + \\
& 64\,a^7\,x^6\,(-1049 + 928\,b\,x) + a^2\,b^4\,(5067 + 4\,b\,x\,(-3147 + 1624\,b\,x)) + \\
& 16\,a^6\,x^4\,(5067 + 4\,b\,x\,(-3147 + 1624\,b\,x)) + \\
& 4\,a^3\,b^2\,(-1629 + b\,x\,(10134 + b\,x\,(-15735 + 6496\,b\,x))) + \\
& 16\,a^5\,x^2\,(-1629 + b\,x\,(10134 + b\,x\,(-15735 + 6496\,b\,x))) + \\
& 4\,a^4\,(261 + 2\,b\,x\,(-3258 + b\,x\,(15201 + 20\,b\,x\,(-1049 + 406\,b\,x))))\big) - \\
& \frac{1}{120}\,e^{-5\,(c+x\,(b+a\,x))}\,t^5\,\big(29525\,b^{10} + 30233600\,a^{10}\,x^{10} + \\
& 10\,a\,b^8\,(-70877 + 59050\,b\,x) + 2560\,a^9\,x^8\,(-70877 + 59050\,b\,x) + \\
& 4\,a^2\,b^6\,(1318286 + 5\,b\,x\,(-567016 + 265725\,b\,x)) + 256\,a^8\,x^6\,(1318286 + \\
& \quad 5\,b\,x\,(-567016 + 265725\,b\,x)) + \\
& 16\,a^3\,b^4\,(-873375 + 2\,b\,x\,(1977429 + 5\,b\,x\,(-496139 + 177150\,b\,x))) + \\
& 256\,a^7\,x^4\,(-873375 + 2\,b\,x\,(1977429 + 5\,b\,x\,(-496139 + 177150\,b\,x))) + \\
& 80\,a^4\,b^2\,(138693 + 2\,b\,x\,(-698700 + b\,x\,(1977429 + 7\,b\,x\,(-283508 + 88575\,b\,x)))) + \\
& 320\,a^6\,x^2\,(138693 + 2\,b\,x\,(-698700 + b\,x\,(1977429 + 7\,b\,x\,(-283508 + 88575\,b\,x)))) + \\
& 32\,a^5\,(-36669 + 10\,b\,x\,(138693 + b\,x\,(-1048050 + b\,x\,(2636572 + 35\,b\,x\,(-70877 + 21258\,b\,x)))))\big)
\end{aligned}$$

Graphs of the solution for $a = b = c = 1$ are shown below.

Graph for the range {x, -1,1}, {t, 0, 1}



Graph for the range {x, -5,5}, {t, 0, 1}



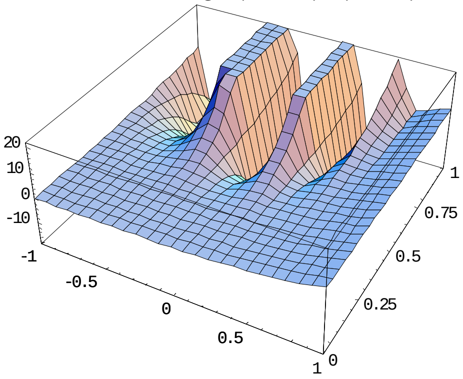
(5-b) $f(u) = e^{-u}$, $g(x) = e^{ax}$

The Adomian solution for general 'a' is

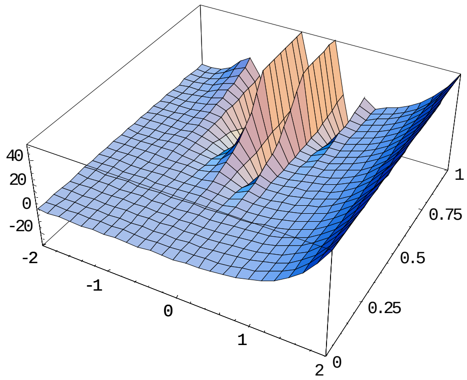
$$
\begin{aligned}
u(x, t) = {} & e^{ax} - a^2 e^{-e^{ax}+ax}\left(-1 + e^{ax}\right) t - \\
& \frac{1}{2} a^4 e^{-2e^{ax}+ax}\left(-1 + 10 e^{ax} - 14 e^{2ax} + 4 e^{3ax}\right) t^2 - \\
& \frac{1}{24} a^8 e^{-4e^{ax}+ax}\left(-1 + 236 e^{ax} - 4304 e^{2ax} + 20192 e^{3ax} - \right. \\
& \left. 35272 e^{4ax} + 26312 e^{5ax} - 8392 e^{6ax} + 928 e^{7ax}\right) t^4 - \\
& \frac{1}{120} a^{10} e^{-5e^{ax}+ax}\left(-1 + 987 e^{ax} - 47374 e^{2ax} + 520713 e^{3ax} - \right. \\
& 2102685 e^{4ax} + 3836499 e^{5ax} - 3477200 e^{6ax} + 1601934 e^{7ax} - \\
& \left. 354385 e^{8ax} + 29525 e^{9ax}\right) t^5 - \\
& \frac{1}{6} a^6 e^{-3e^{ax}+ax} t^3 \left(-1 + e^{3ax}\left(515 - 597 \operatorname{Cosh}[a x] + 98 \operatorname{Cosh}[2 a x] + 39 \operatorname{Sinh}[a x] - 8 \operatorname{Sinh}[2 a x]\right)\right)
\end{aligned}
$$

The behavior of the solution for $a = 2$ is shown by the following graphs.

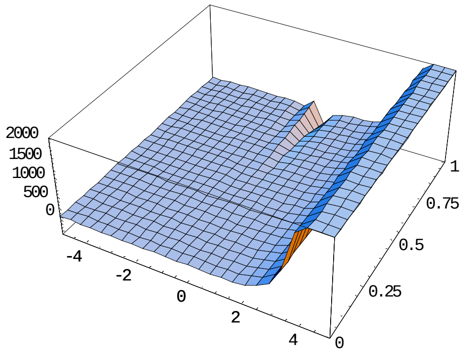
Range {x, -1,1}, {t, 0, 1}



Range {x, -2,2}, {t, 0, 1}

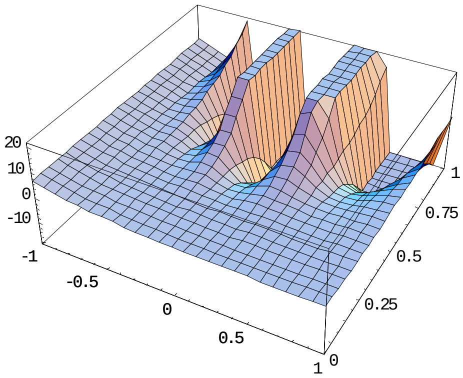


Range {x, -5,5}, {t, 0, 1}

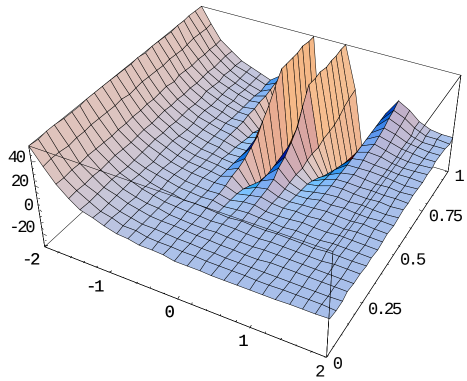


Some graphs of the solution for $a = -2$ are

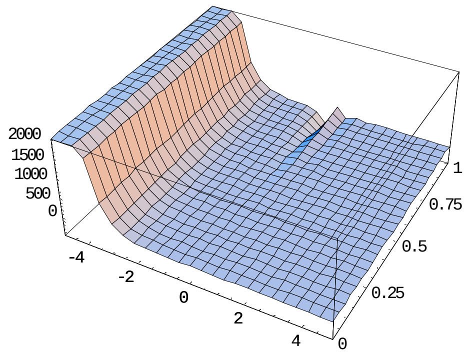
Range {x, -1,1}, {t, 0, 1}



Range {x, -2,2}, {t, 0, 1}



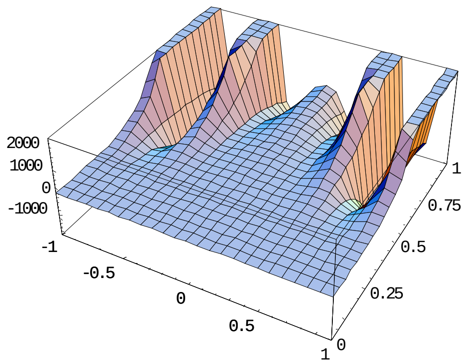
Range {x, -5,5}, {t, 0, 1}



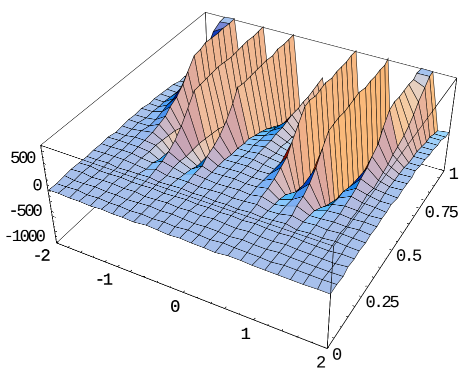
(5-c) $f(u) = e^{-u}$, $g(x) = e^{-ax^2}$

Solution for general 'a' can be obtained from authors as Mathematica file. The nature of the solution for $a = 2$ is shown by the following graphs.

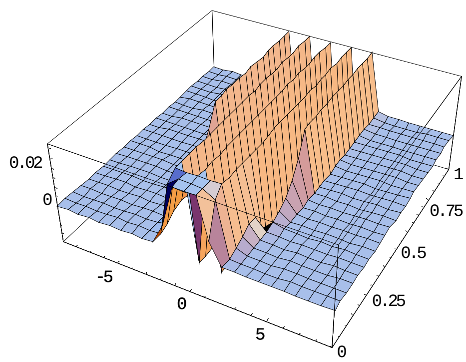
Range {x, -1,1}, {t, 0, 1}



Range {x, -2,2}, {t, 0, 1}



Range {x, -9,9}, {t, 0, 1}



(5-d) $f(u) = e^{-u}$, $\quad g(x) = \text{sech}^2 x$

The Adomian solution for this case is

$$u(x,t) = \text{Sech}[x]^2 + e^{-\text{Sech}[x]^2}\, t\, (-3 + \text{Cosh}[2x])\, \text{Cosh}[2x]\, \text{Sech}[x]^6 +$$

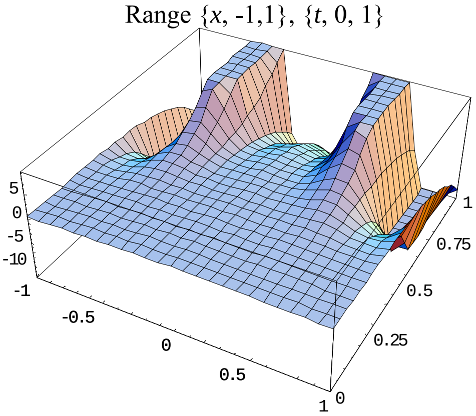
$$\frac{1}{3072}\Big(e^{-3\,\text{Sech}[x]^2}\, t^3\,(221115 - 397686\,\text{Cosh}[2x] + 368772\,\text{Cosh}[4x] - 220146\,\text{Cosh}[6x] + 175684\,\text{Cosh}[8x] - 80214\,\text{Cosh}[10x] + 10620\,\text{Cosh}[12x] - 322\,\text{Cosh}[14x] + \text{Cosh}[16x])\,\text{Sech}[x]^{18}\Big) +$$

$$\frac{1}{196608}\Big(e^{-4\,\text{Sech}[x]^2}\, t^4\,(-240500624 + 480947158\,\text{Cosh}[2x] - 399748784\,\text{Cosh}[4x] + 333253082\,\text{Cosh}[6x] - 235333184\,\text{Cosh}[8x] + 134853173\,\text{Cosh}[10x] - 87698360\,\text{Cosh}[12x] + 29901923\,\text{Cosh}[14x] - 3766832\,\text{Cosh}[16x] + 152503\,\text{Cosh}[18x] - 1432\,\text{Cosh}[20x] + \text{Cosh}[22x])\,\text{Sech}[x]^{24}\Big) +$$

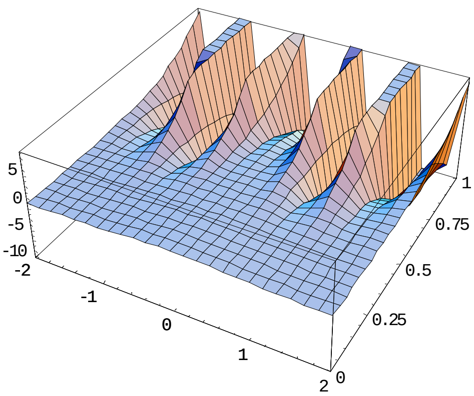
$$\frac{1}{15728640}\Big(e^{-5\,\text{Sech}[x]^2}\, t^5\,(444450799780 - 852362485688\,\text{Cosh}[2x] + 765886466779\,\text{Cosh}[4x] - 647640107522\,\text{Cosh}[6x] + 478419845370\,\text{Cosh}[8x] - 359156338570\,\text{Cosh}[10x] + 212556563257\,\text{Cosh}[12x] - 119102101972\,\text{Cosh}[14x] + 63026707068\,\text{Cosh}[16x] - 17871306804\,\text{Cosh}[18x] + 2197172715\,\text{Cosh}[20x] - 106796390\,\text{Cosh}[22x] + 1721318\,\text{Cosh}[24x] - 5966\,\text{Cosh}[26x] + \text{Cosh}[28x])\,\text{Sech}[x]^{30}\Big) + \frac{1}{16}\, e^{-2\,\text{Sech}[x]^2}\, t^2\,(255 + 90\,\text{Cosh}[2x] + 288\,\text{Cosh}[4x] - 58\,\text{Cosh}[6x] + \text{Cosh}[8x])\,\text{Sech}[x]^{10}\,\text{Tanh}[x]^2$$

The behavior of the solution can be understood from the following five graphs.

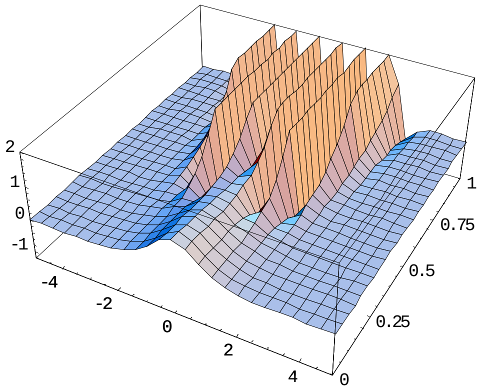
Range {x, -1,1}, {t, 0, 1}



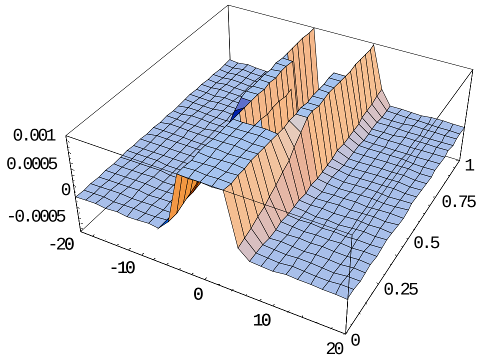
Range {x, -2,2}, {t, 0, 1}



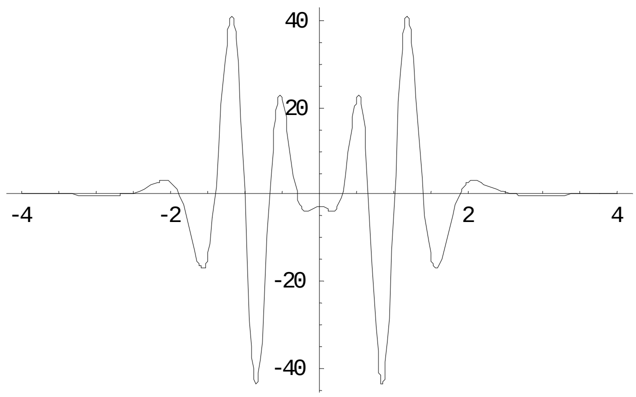
Range {*x*, -5,5}, {*t*, 0, 1}



Range {*x*, -20,20}, {*t*, 0, 1}



Graph for fixed t=0.8 for the range {*x*, -4,4}



4. Conclusion

Adomian decomposition method is applied to investigate solutions of several cases of non-linear heat equation using different initial conditions. The method is quite efficient to determine solutions in fast converging power series. It is also useful for closed form solutions when they exist. However, most methods of calculating Adomian polynomials require computational formulas and power that make them difficult to implement algorithmically using software. We have implemented the method of [14] for calculating Adomian polynomials and used it to find solutions of many cases of non-linear heat equation with different types of initial conditions. The algorithm can be used without any complex calculations and only involves elementary operations.

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