King Fahd University of Petroleum & Minerals

DEPARTMENT OF MATHEMATICAL SCIENCES

Technical Report Series

TR 213

December 1996

History Effects on Oil Recovery Efficiency

Khaled M. Furati

DHAHRAN 31261 ● SAUDI ARABIA ● www.kfupm.edu.sa/math/ ● E-mail: mathdept@kfupm.edu.sa

History Effects on Oil Recovery Efficiency

Khaled M. Furati

Department of Mathematical Sciences

King Fahd University of Petroleum and Minerals

Dhahran 31261, Saudi Arabia

kmfurati@dpc.kfupm.edu.sa

Abstract

In this paper we illustrate the possible effects of saturation history in enhanced oil recovery through water and polymer flooding. We present scenarios under which the flow history might have a drastic impact on the flow and efficiency prediction. Favorable recoveries can be achieved via adjusting the preinjection conditions of reservoirs.

Keywords: enhanced oil recovery, efficiency, waterflooding, history, fluid injection.

1 Introduction

For rocks exhibiting strong wettability preference for a specific phase, experiments have shown that flow characteristics depend on whether the wetting or nonwetting phase saturation is increasing. The flow is called *drainage* or *imbibition* in reference to saturation change of the wetting phase. The flow dependence on saturation direction is contributed largely to nonwetting phase entrapment and lead to developing flow properties for imbibition as well as drainage flows.

For determining the flow characteristics, appropriate cycles of drainage-imbibition or imbibition-drainage processes should be conducted to obtain relations for possible intermediate flows. These relations are essential for more accurate flow prediction through mathematical models. Mathematically, this results in flow functions which are not only functions of current fluid saturation but also of saturation history.

One of the flow characteristics that plays a crucial role in flow prediction is the relative permeability. Relative permeability relations should reflect the saturation direction effects for imbibition and drainage flows. As a result, relative permeability should be a function of saturation as well as saturation history that describes the way that saturation is approached.

In (Furati, b) the author analyzed and solved a history dependent three-component two-phase model for polymer flooding. In (Furati, a) we considered a more general model for the history dependence of relative permeability and constructed the Riemann problem for the conservation law describing the immiscible two-phase flow in porous media. Although the saturation profile for the flow In (Furati, a) is monotone, we found that the flow history has a notable impact on breakthrough times and compositions for displacement processes.

In this paper we investigate the flow history effects on polymer flooding. In particular, we focus on conditions under which the predicted saturation profile is nonmonotone and gives rise to oil banks. We present examples that illustrate the further structural changes in fluid distribution. In addition, we show the possible effects of flow history on effluent history and oil recovery efficiency.

In Section 2 we describe briefly the history dependent mathematical model. In Section 3 we interpret the model for oil recovery by polymer flooding. In Section 4 we examine the monotonicity of the saturation profile. In Section 5 we look at the effluent history and recovery efficiency. In Section 6, we present examples for flow history effects.

2 Mathematical Model

In this section we summarize the work done in (Furati, a) and (Furati, b). Polymer flooding can be modeled as a two-phase three-component flow. The two phases are the aqueous and the oleic phase. The components are water, polymer and oil. The polymer coexists only with water.

A one-dimensional immiscible two-phase flow with a constant total velocity can be described mathematically under reasonable assumptions by the following dimensionless conservation laws

$$\partial_t S + \partial_x F = 0, \qquad (1)$$

$$\partial_t (C\,F) + \partial_x (C\,F) = 0,$$

where S and F are the saturation and fractional flow function, respectively, for the aqueous phase. The variable C is the polymer saturation.

To derive (1) we assume that the porous medium is homogeneous and the phases are incompressible and in local equilibrium. We consider the aqueous phase viscosity to be an increasing function of polymer concentration while the oleic phase viscosity to be constant. Moreover, we assume that diffusive forces and gravity effect are negligible. For more details on the derivation of (1) and physical assumptions see (Pope, 1980).

In any part of the porous medium, we consider the flow to be *primary* if the displacement process corresponds to continuously displacing a phase completely fills the pores initially. If the displaced phase is the wetting phase we call the flow *primary drainage*. However, if the displaced phase is the nonwetting phase we call the flow *primary imbibition*.

If a flow reversal occurs during a primary flow, we call the flow a *secondary flow*. This includes the secondary drainage of a primary imbibition (id) and secondary imbibition of a primary drainage (di). We denote the critical saturation at which the flow reversal occurs by S_h, where $h = id, di$, indicates the relevant flow cycle for the secondary flow.

In this study we consider the aqueous phase as the wetting phase. Accordingly, the history dependent fractional flow function F of the wetting phase takes the form

$$F = \begin{cases} F^p(S,C) & \text{for primary flow,} \\ F^h(S,S_h,C) & \text{for secondary flow.} \end{cases} \tag{2}$$

Please see (Furati, a) for typical fractional flows. Note that adding polymer to the water increases water viscosity. Thus, the fractional flow function is a decreasing function of the polymer concentration C.

The conservation law (1) requires initial data at each point in space. We consider the Riemann initial conditions

$$(S,S_h,C)(x,0) = \begin{cases} (S^L,S_h^L,C^L) & x \leq 0, \\ (S^R,S_h^R,C^R) & x > 0, \end{cases} \tag{3}$$

where, for the primary flow, the initial condition (3) reduces to values for S and C only, since $S_h = S$. If the flow is secondary, then the conservation law (1) can be closed by adding the constraint $\partial_t S_h = 0$ for the reversibility assumption. The solution of (1)–(3) describes the saturation distribution and history at each point of space and time.

Solutions of the Riemann problem for the system (1) consist of self-sharpening fronts (*shocks*), expansion waves (*rarefactions*) and contact discontinuities. These waves can be associated with the appropriate waves families of each type of flow.

For the primary flow, there are two wave families. The first one is associated with the characteristic speed $\partial_S F^p$ and consists of rarefactions and shocks. This family is in general not genuinely nonlinear and across the corresponding waves the polymer concentration C is constant. The second wave family is linearly degenerate and associated with the particle velocity F^p/S. Thus, this family consists only of contact discontinuities that travel with speed equals to F^p/S.

For the secondary flow case, there are analog wave families. Across the waves of these families the history value S_h is constant. Moreover, the solution may start with a stationary contact discontinuity from the left state across which the fractional flow is constant. This stationary contact discontinuity is an outcome of the reversibility assumption on the flow ($\partial_t S_h = 0$).

Physically, this discontinuity acts as an adjustment for the saturation to keep the same rate of flow for two adjacent parts of the porous medium with different histories.

Graphically, the solution of the Riemann problem can be found by constructing a horizontal line from the left state, appropriate hulls for the related fractional flow functions, and chords from $(0,0)$ to some states. These components are controlled by the compatibility condition for the waves speed.

3 Oil Recovery

The Riemann problem for system (1) can be used to predict flows that occur in water and polymer flooding techniques used in enhanced oil recovery. In polymer flooding, oil-free polymer solution is injected into the oil reservoir to drive out oil.

The particular stage in the producing life of a reservoir determines the preinjection saturation and history of the reservoir. There are three stages for oil production: primary, secondary and tertiary recovery. Primary recovery is oil recovery by natural drive mechanisms. Secondary recovery refers to techniques whose purpose, in part, is to maintain reservoir pressure. Water injection is an example of secondary recovery. Tertiary recovery is any technique applied after secondary recovery.

We denote the preinjection fluid composition in a porous medium by (S^I, S_h^I, C^I). The initial saturation history S_h^I depends on the sequence of events to which the porous medium was exposed or through which it went. As in (Furati, a), we consider two scenarios for the preinjection flow in a porous medium.

The first scenario is to assume that the porous medium has been undergoing either a primary drainage or imbibition. In this case the saturation value and the critical saturation for the history are equal, i.e., $S^I = S_h^I$. The preinjection flow characteristics in this case are extracted from the primary fractional flow curve.

The second scenario is to assume that the porous medium under consideration was exposed to an event of flow reversal at some saturation S_h^I. In this case the preinjection flow characteristics are determined by the secondary fractional flow curve associated with S_h^I.

Note that the injected fluid is aqueous and thus the injected saturation is high. We assume that for the injected state the Buckley-Leverett speed is slower than the particle velocity, i.e., $\partial_S F < F/S$. Moreover, we assume that the reservoir preinjection composition is polymer free.

For water flooding, as in (Furati, a), the saturation profile is monotone. As a result, at any location in the reservoir between the injecting and producing well, there are three possible displacement flows that might take place during the flooding. The flow might be completely primary, completely secondary or secondary then becomes primary.

For polymer flooding described by imbibition-drainage fractional flow curves the same behavior occurs depending on the preinjection flow in the porous medium. If before the flooding, the porous medium went through primary imbibition that resulted in the irreducible oil saturation, then the flow caused by the flooding at any position in the reservoir is secondary. Otherwise, the flow during the flooding at any position is either primary, or secondary then becomes primary. This is the case whether the saturation profile is monotone or not as shown in (Furati, b).

For polymer flooding described by drainage-imbibition fractional flow curves, let S^q be the saturation such that the chord from the origin to $(S^q, F^d(S^q, 0))$ is tangent to the curve $F^{di}(S, S^q, C^J)$ at some point. Else, let S^q be the saturation such that the chord and the curve intersects only when $F = F^J$ as shown in Figure 1. The existence of S^q follows from the continuity of the functions. If the preinjection saturation is greater than S^q and a result of primary drainage then the flooding produces an oil bank and the flow at any position starts as primary then switches to secondary. Otherwise, the flow is secondary even if the saturation profile is nonmonotone. Please see Figure 10 and 11.

To sum up, flows predicted by the history dependent model (1)–(3) may result in monotone or nonmonotone profiles which may or may not be accompanied by flow type change. Therefore,

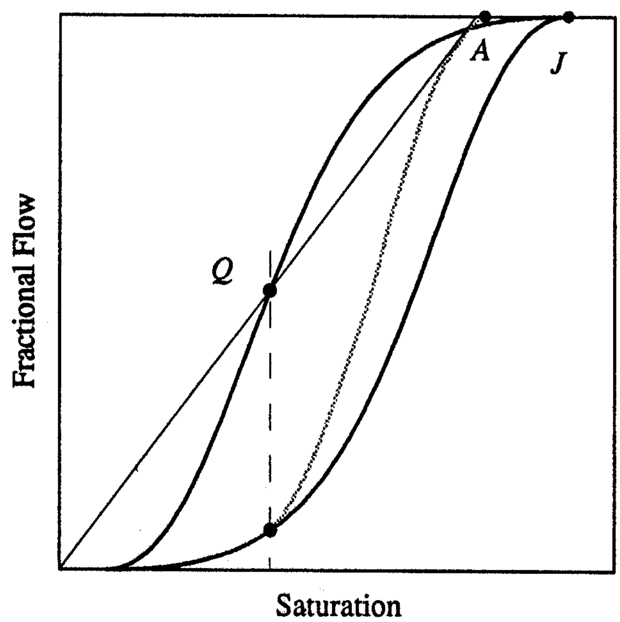


Figure 1: The critical states for drainage-imbibition flow.

the primary and appropriate secondary curves are needed for the flow prediction. Otherwise, one set of the curves is sufficient to describe the flow.

Note that the saturation profile has a jump that corresponds to the *polymer solution front.* Behind the front (upstream) the aqueous phase contains polymer of concentration C^J. Ahead of the front (downstream) the aqueous phase is polymer free.

4 Saturation Profile Monotonicity

Typically, the reservoir initial aqueous phase saturation is low and close to the irreducible saturation after primary recovery. However, the initial saturation can be higher for those reservoirs that had already been waterflooded for secondary recovery. The high level of the preinjection aqueous phase saturation in polymer flooding may lead to nonmonotone distribution of the fluids during the recovery process even if the history effects are not considered.

To illustrate the possible oil displacement scenarios we define several critical states associated with the injected composition U^J and the initial composition U^I. Let $U^T = (S^T, S^T, C^J)$, $U^B = (S^B, S^B, 0)$ and $U^M = (S^M, S^M, 0)$ be the states shown in Figure 2. Note that S^B can be

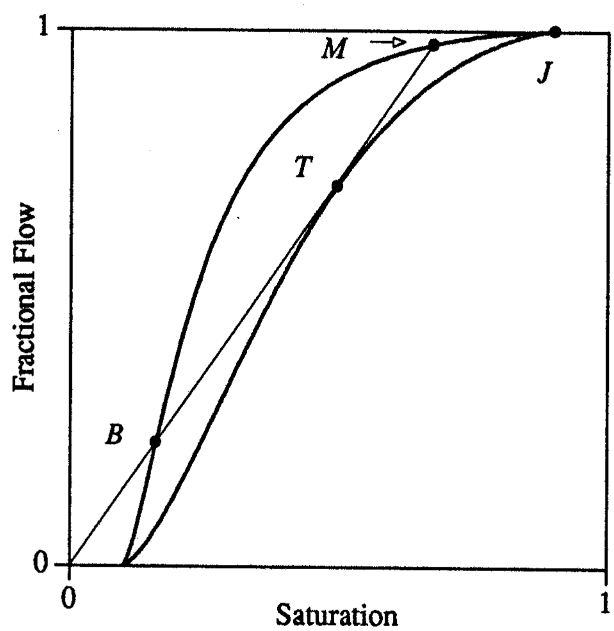


Figure 2: The critical states for polymer flooding.

remarkably high for some fractional flows and S^M can be low for others.

When $S^I \leq S^B$ the saturation profile is monotone as shown in Figure 3. In this case the saturation at any fixed position in the reservoir increases with time. Therefore, only one direction fractional flow curves set is needed for predicting the flow. On the other hand, if the initial saturation of an oil reservoir, S^I, exceeds S^B then the saturation profile is not monotone. For such a displacement, secondary fractional flow curves for the appropriate flow reversals are needed. When history dependence is not taken into account, there are two possible nonmonotone profiles resulting in two interesting scenarios:

1. Oil Bank Profile ($S^B < S^I \leq S^M$)

When S^I lies between S^B and S^M, the flow develops a polymer-free region of high oil saturation followed by a polymer solution front. In petroleum engineering literature, this is called an *oil bank*. Please see Figure 4. This flow is preferable since a substantial amount of oil is recovered in a relatively short time.

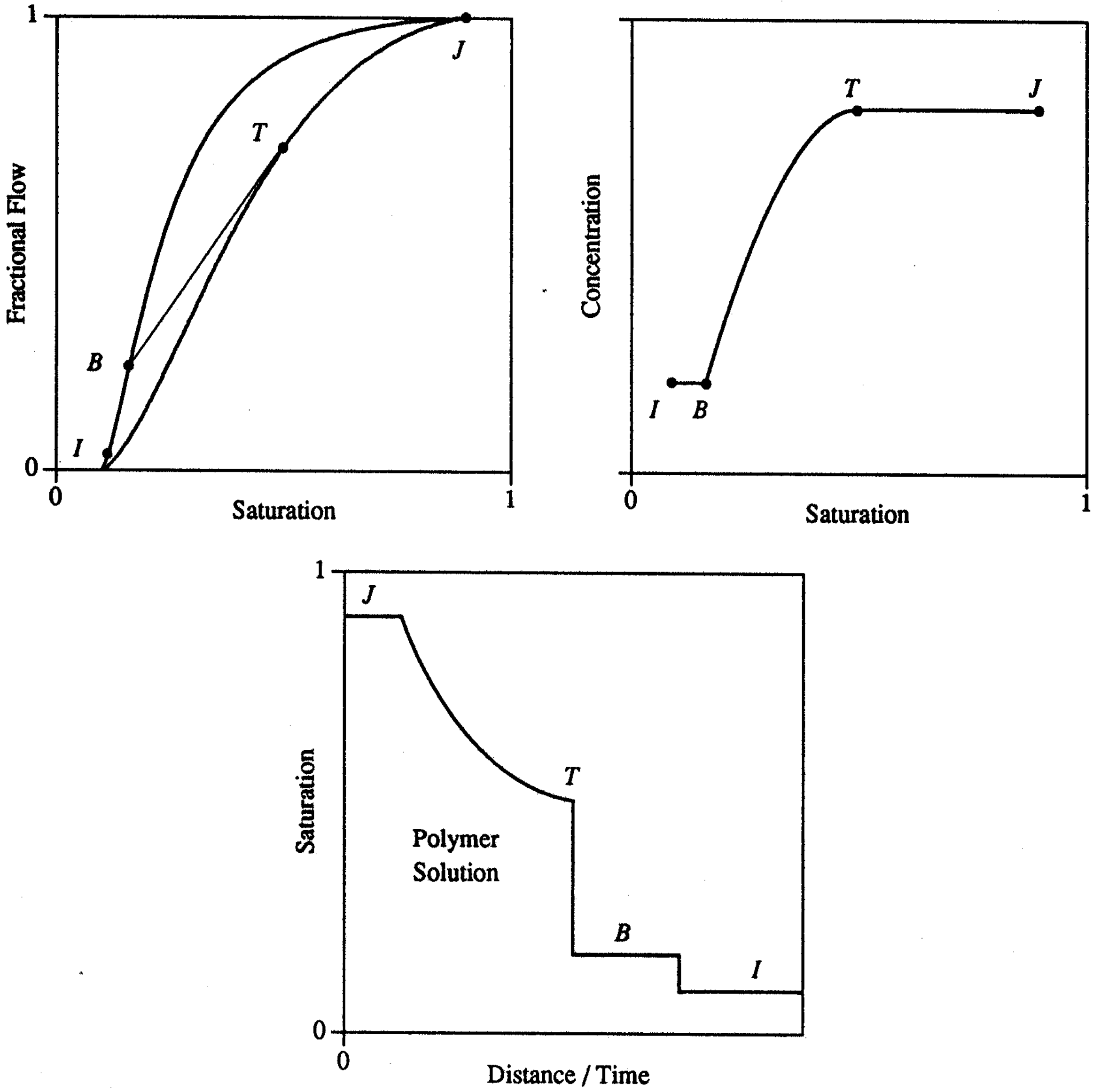


Figure 3: Monotone profile.

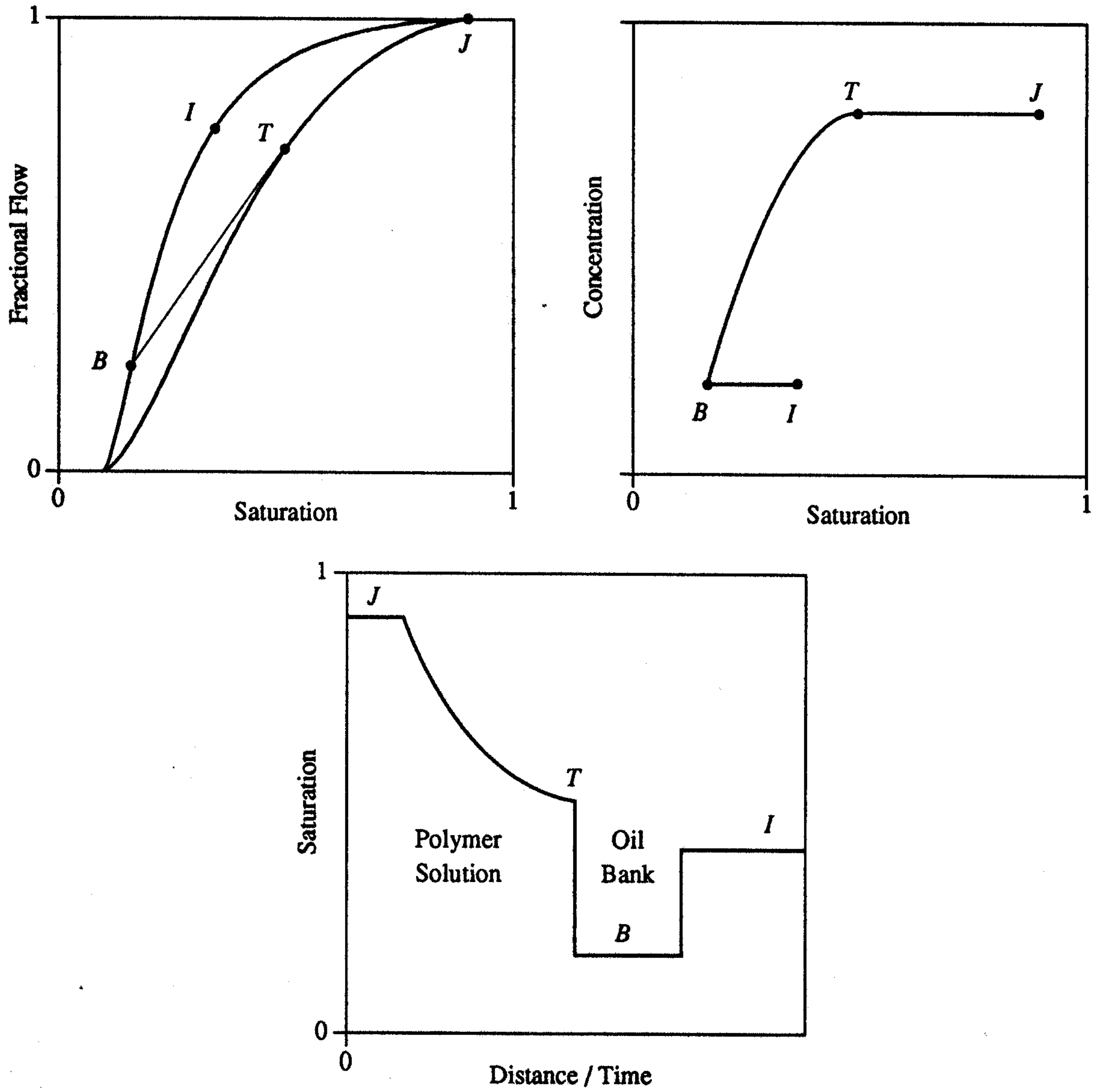


Figure 4: Oil banks profile.

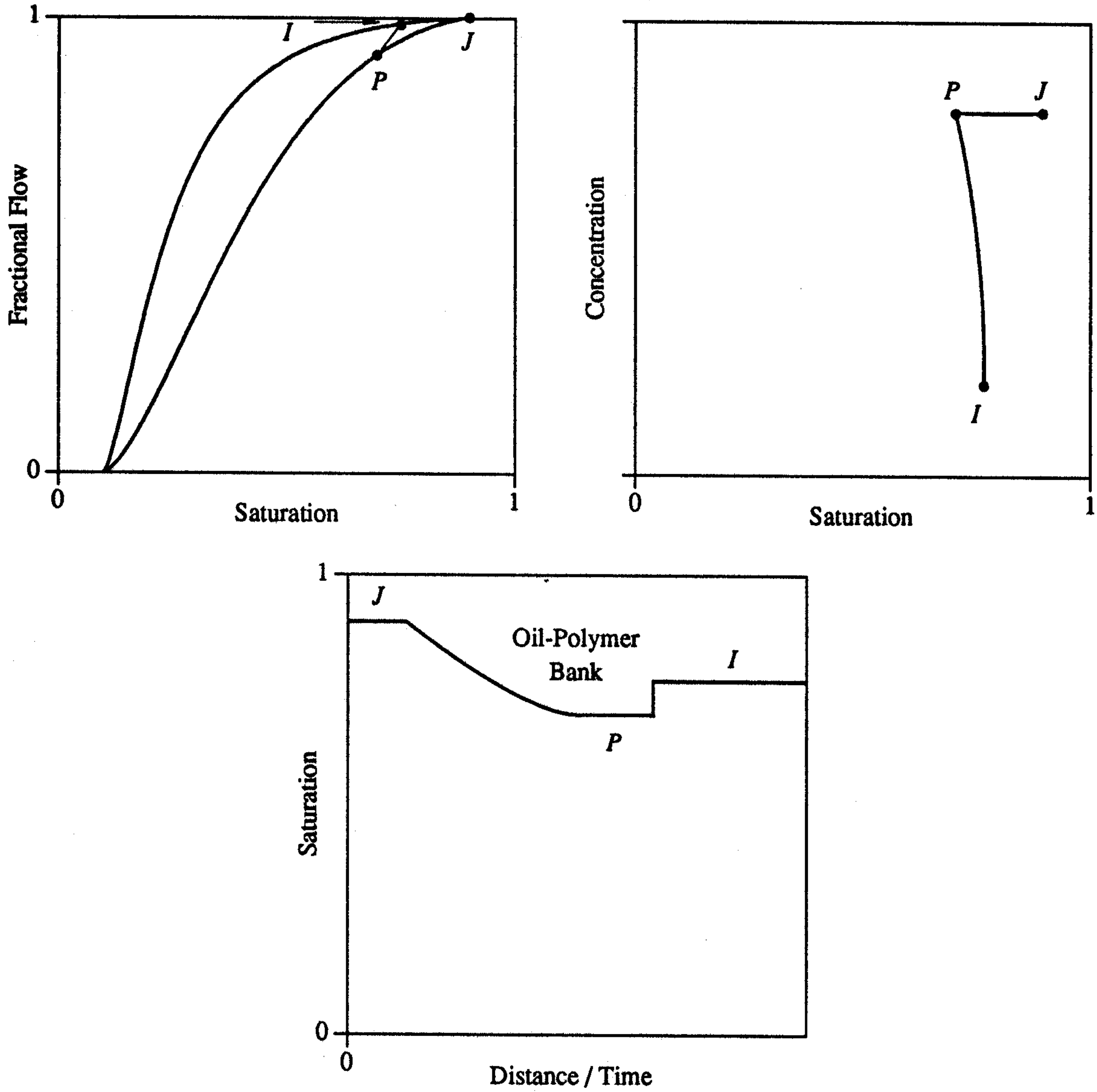


Figure 5: Oil-polymer bank profile.

2. Oil-Polymer Bank Profile ($S^M < S^I$)

When S^I exceeds the critical saturation S^M, the polymer solution front takes over the oil bank. This leads to the formation of a region of high oil saturation coexisting with polymer solution. We use the term *oil-polymer bank* to refer to such a region. An oil-polymer bank is shown in Figure 5. Note that this flow is less desirable for two reasons: low oil bank saturation and polymer solution existence.

5 Effluent History and Recovery Efficiency

There are two predictions that are helpful in designing an oil recovery process: ***effluent history*** and ***recovery efficiency.***

Effluent history is the oil saturation at the producing well as a function of time. Note that for the two flows with banks described above there are two important events: oil bank and polymer solution breakthrough. After the oil bank breakthrough, the well produces the maximum possible amount of oil. After the polymer solution breakthrough the well produces some of the solution being injected. We mark the breakthrough times as follows:

$$\begin{aligned} t^{ob} &= \text{oil bank breakthrough time,} \\ t^{ps} &= \text{polymer solution breakthrough time.} \end{aligned} \tag{4}$$

We use the subscript h to refer to the same incidents for the history dependent model. For efficiency, we would like to conduct the displacement so that t^{ob} is as small as possible and t^{ps} is as large as possible.

The recovery efficiency at time t, $E(t)$, is defined in (Lake, 1989) by

$$E = \frac{\text{Amount of oil displaced}}{\text{Initial amount of oil in the reservoir}}. \tag{5}$$

For the oil recovery process described by (1) and (3) the the recovery efficiency reduces to

$$E(t) = \frac{[1 - S^I] - [1 - \bar{S}(t)]}{1 - S^I} = \frac{\bar{S}(t) - S^I}{1 - S^I}, \tag{6}$$

where $\bar{S}(t)$ is the average aqueous phase saturation in the reservoir at time t,

$$\bar{S}(t) = \int_0^1 S(x,t)\,dx. \tag{7}$$

Recall that the speed of any traveling jump in a saturation profile is equal to the slope of the chord connecting the adjacent states. Therefore, the position X^j of a jump connecting a left and right state at time t is given by

$$X^j = \left\{ \frac{F^R - F^L}{S^R - S^L} \right\} t. \tag{8}$$

In a rarefaction (spreading) part of the flow all the flux variables are constant except saturation. Thus, along rarefactions we have $\partial_x F = \partial_S F \, \partial_x S$. Moreover, the position of an intermediate saturation $S(t)$ at time t is given by $t \, \partial_S F(S(t))$. Similarly, at any fixed position, we have $\partial_t F = \partial_S F \, \partial_t S$ during rarefactions.

From the above, given a saturation profile $S(x,t)$ for a Riemann problem solution of (1), then

$$\int_a^b S(x,t)\, dx = b\, S^{out} - a\, S^{in} + t\, \left[F^{in} - F^{out}\right], \tag{9}$$

where the superscripts correspond to the flow in and out of the segment (a,b). In particular,

$$\bar{S}(t) = S^{out}(t) + t\,[F^J - F^{out}(t)]. \tag{10}$$

The rate of change of the recovery efficiency is given by

$$\begin{aligned} D_t E(t) &= \frac{1}{1-S^I}\, \left[(1 - t\,\partial_S F)\, D_t S^{out}(t) + F^J - F^{out}(t)\right] \\ &= \frac{1}{1-S^I}\, \left[F^J - F^{out}(t)\right] > 0, \end{aligned} \tag{11}$$

since $t\,\partial_S F = 1$. This shows that $E(t)$ is an increasing function and is linear for constant outflow saturations. Also, during the outflow of a spreading saturation profile, $E(t)$ is concave since

$$D_{tt}E(t) = (-\partial_S F\, \partial_t S)/(1 - S^I) < 0. \tag{12}$$

6 Flow History Effects

To illustrate the saturation history effects on the flow and recovery efficiency prediction, consider the examples depicted in Figures 6–9 for the imbibition-drainage model.

Figure 6 shows the case when the initial reservoir saturation S^I lies between S^B and S^T. Note that for the history dependent model the oil bank is faster but with lower oil saturation. Moreover, ignoring the history dependence underestimates the recovery efficiency.

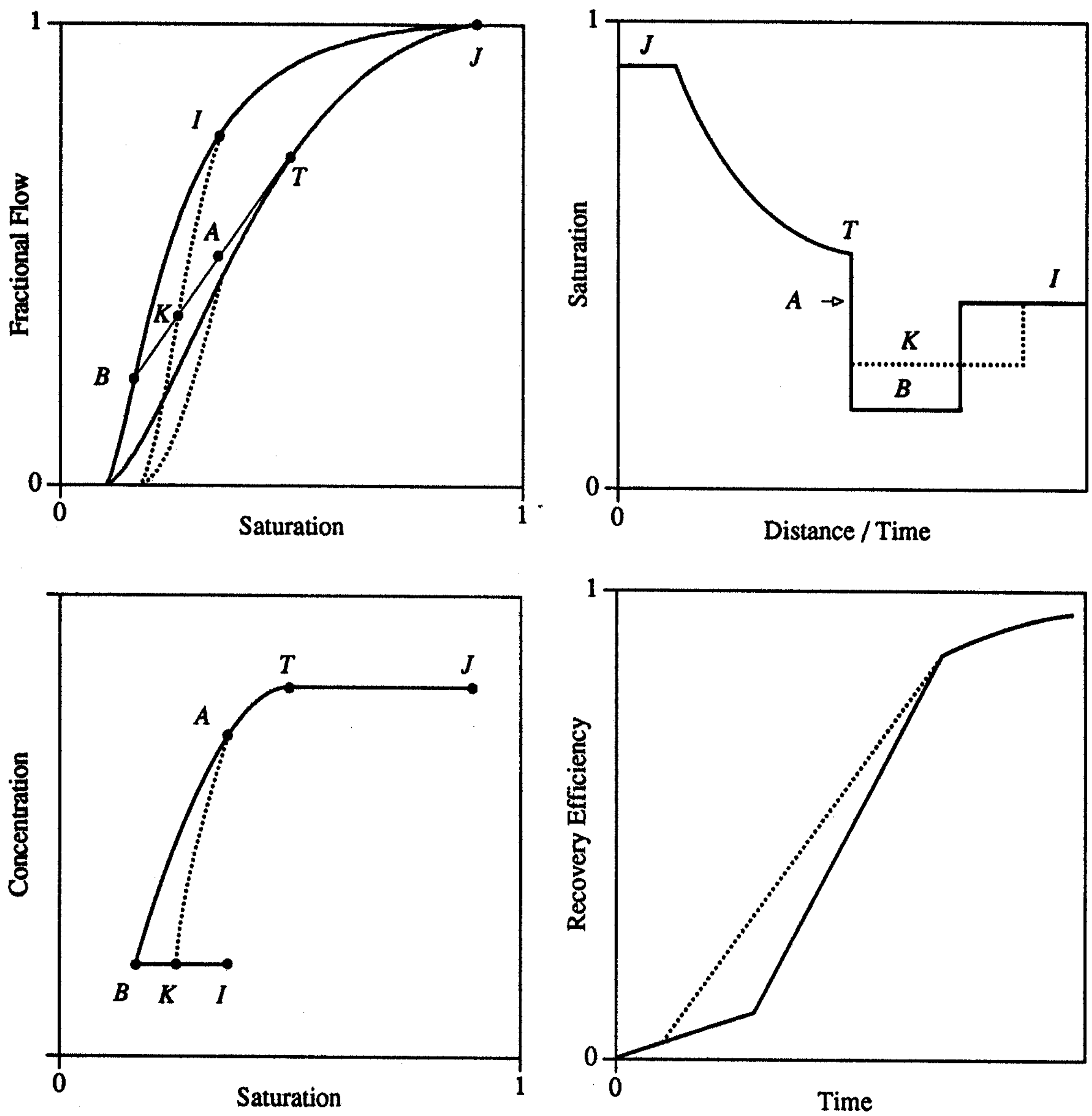
Figure 7 corresponds to the case $S^T < S^I \leq S^M$. Note that the saturation history produces a wider bank but with lower oil saturation. This improves the recovery efficiency in the short run.

Figures 8 and 9 describe the flooding of reservoirs with high initial water saturation, $S^M < S^I$. As shown in Figure 5, the flow develops an oil-polymer bank. However, saturation history has two remarkable impacts on the flow prediction. First, it increases the oil saturation of the bank while shortening its recovery time as in Figure 8. Second, saturation history might delay the polymer solution breakthrough and advances the oil bank breakthrough as in Figures 9. As a result, a polymer-free oil bank develops. Again, the recovery efficiency is higher for the early stages of the recovery.

Finally, similar effects are observed for the drainage-imbibition model as shown in Figure 10 and 11. Note that for the case in Figure 10, ignoring the flow history results in overestimating the recovery efficiency.

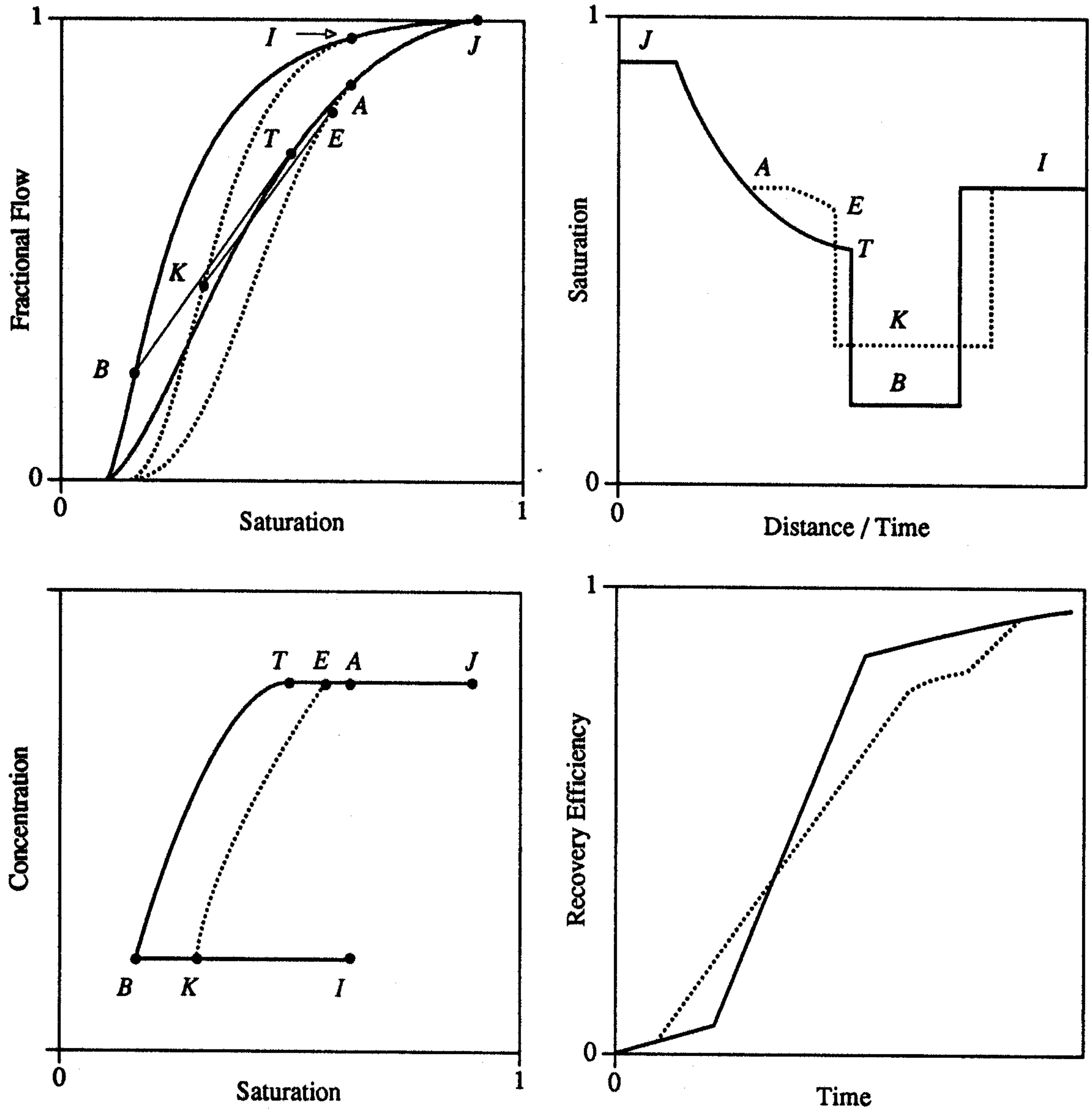
Acknowledgment

The author is grateful for the financial support provided by King Fahd University of Petroleum and Minerals.



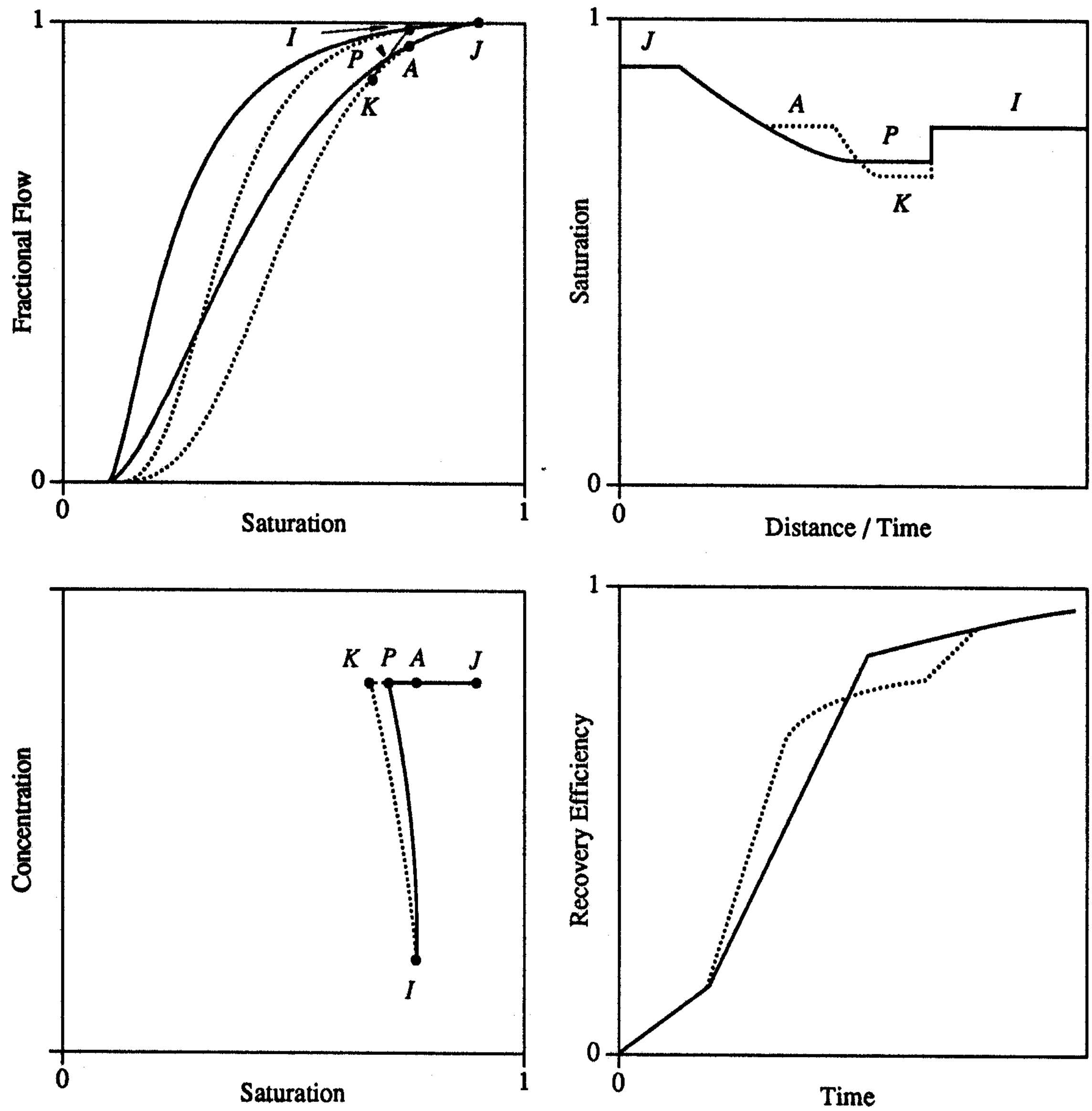
t	t^{ob}	t^{ps}	
$\bar{S}(t)$	$S^I + t\,[F^J - F^I]$	$S^B + t\,[F^J - F^B]$	$S(t) + t\,[F^J - F(t)]$
$\bar{S}_h(t)$	$S^I + t\,[F^J - F^I]$	$S^K + t\,[F^J - F^K]$	$S(t) + t\,[F^J - F(t)]$
t_h	t_h^{ob}	t_h^{ps}	

Figure 6: History effects on oil bank for imbibition-drainage model.



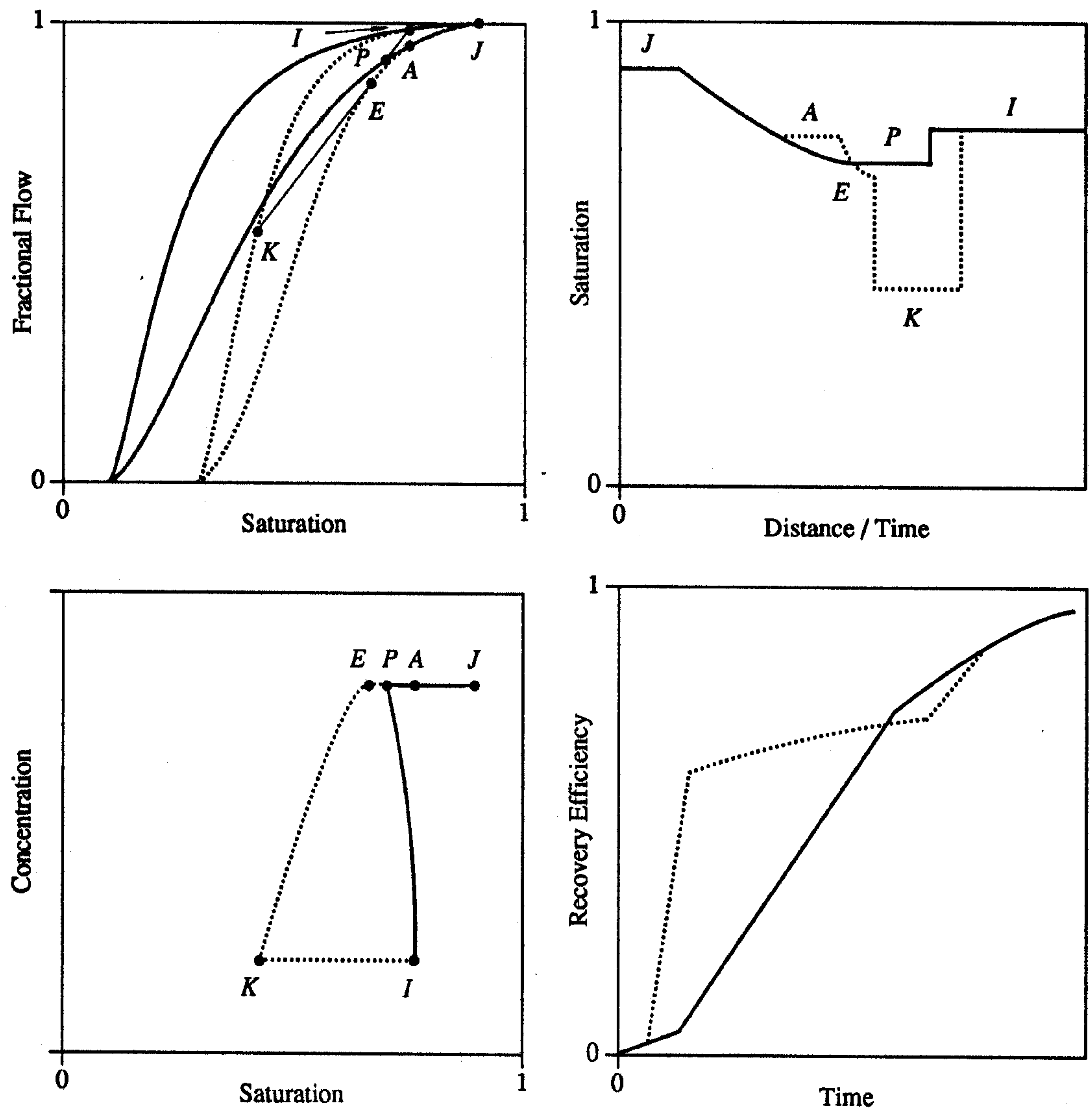
<table>
<tr><td>t</td><td colspan="2">t^{ob}</td><td>t^{ps}</td><td colspan="4"></td></tr>
<tr><td>$\bar{S}(t)$</td><td colspan="2">$S^I + t\,[F^J - F^I]$</td><td>$S^B + t\,[F^J - F^B]$</td><td colspan="4">$S(t) + t\,[F^J - F(t)]$</td></tr>
<tr><td>$\bar{S}_h(t)$</td><td>$S^I + t\,[F^J - F^I]$</td><td colspan="3">$S^K + t\,[F^J - F^K]$</td><td>$S(t) + t\,[F^J - F(t)]$</td><td>$S^A + t\,[F^J - F^A]$</td><td></td></tr>
<tr><td>t_h</td><td>t_h^{ob}</td><td colspan="3">t_h^{ps}</td><td></td><td></td><td></td></tr>
</table>

Figure 7: History effects on oil bank for imbibition-drainage model.



t	t^{ps}			
$\bar{S}(t)$	$S^I + t\,[F^J - F^I]$	$S^P + t\,[F^J - F^P]$	$S(t) + t\,[F^J - F(t)]$	
$\bar{S}_h(t)$	$S^I + t\,[F^J - F^I]$	$S^K + t\,[F^J - F^K]$	$S(t) + t\,[F^J - F(t)]$	$S^A + t\,[F^J - F^A]$
t_h	t_h^{ps}			

Figure 8: History effects on oil-polymer bank for imbibition-drainage model.



t	t^{ps}			
$\bar{S}(t)$	$S^I + t[F^J - F^I]$	$S^P + t[F^J - F^P]$	$S(t) + t[F^J - F(t)]$	
$\bar{S}_h(t)$	$S^I + t[F^J - F^I]$	$S^K + t[F^J - F^K]$	$S(t) + t[F^J - F(t)]$	$S^A + t[F^J - F^A]$
t_h	t_h^{ob}	t_h^{ps}		

Figure 9: History effects on oil-polymer bank for imbibition-drainage model.

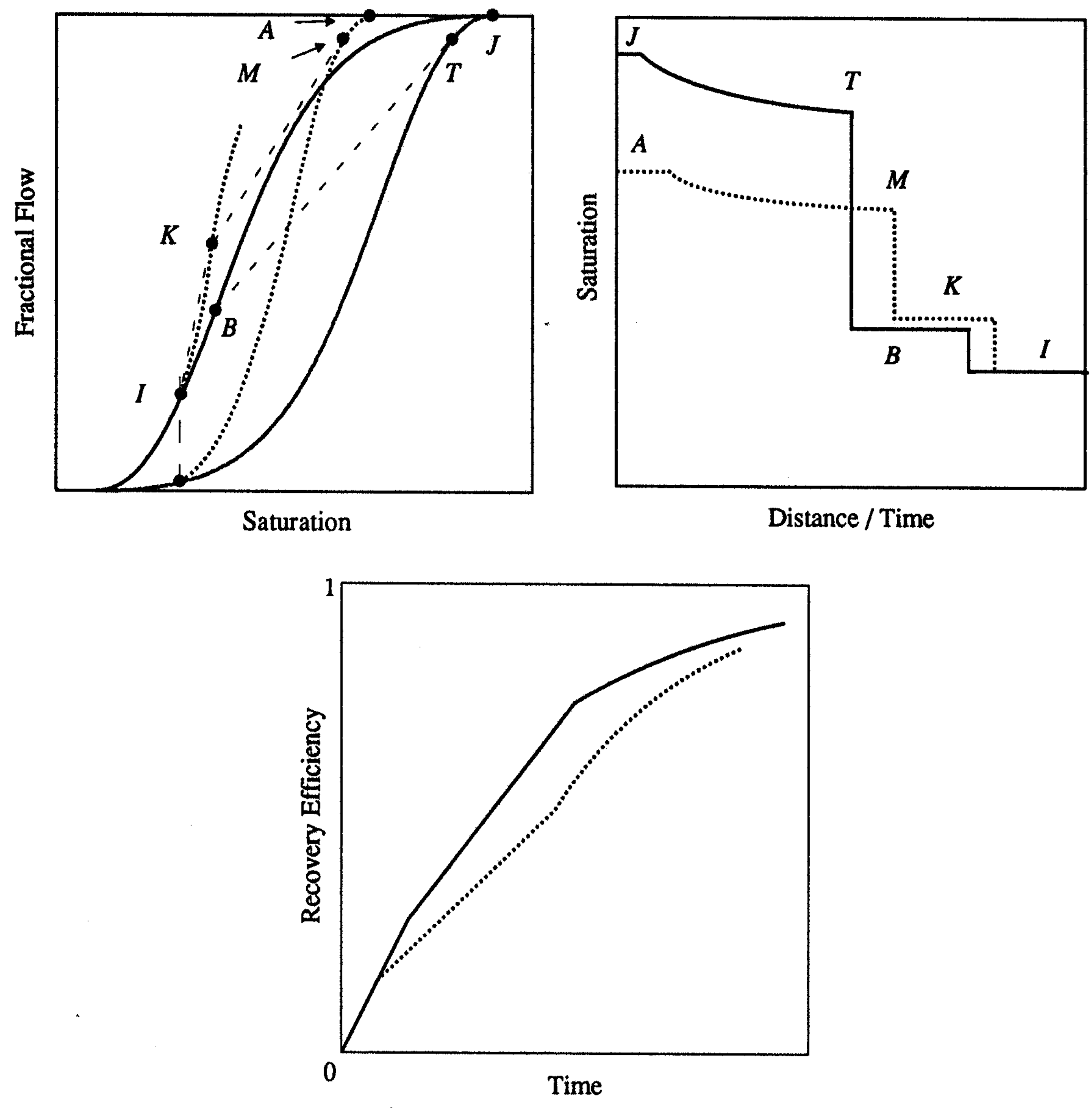
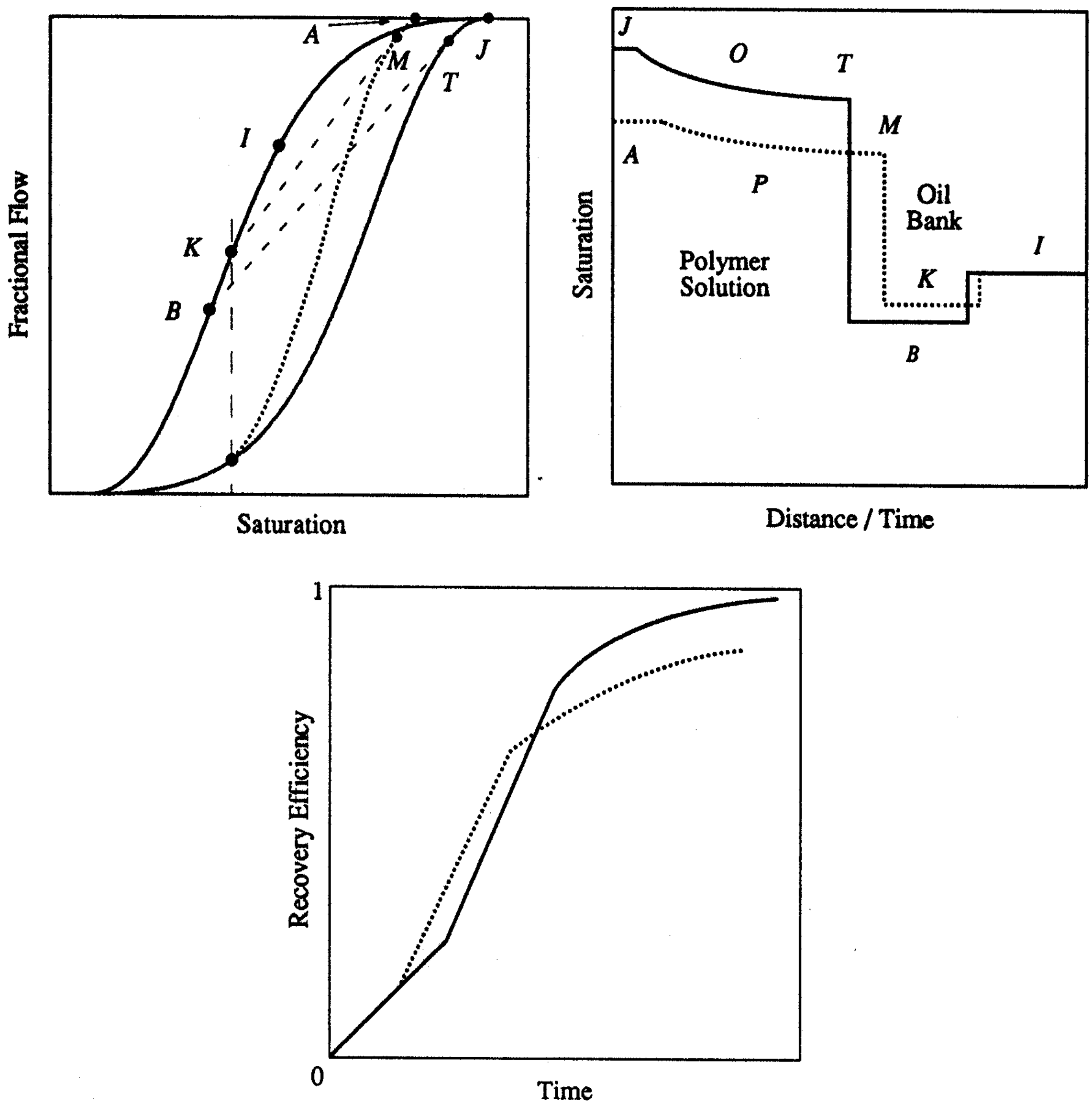


Figure 10: History effects on monotone profile for the drainage-imbibition model.



t	t^{ob}	t^{ps}	
$\bar{S}(t)$	$S^I + t[F^J - F^I]$	$S^B + t[F^J - F^B]$	$S^O(t) + t[F^J - F^O)$
$\bar{S}_h(t)$	$S^I + t[F^J - F^I]$	$S^K + t[F^J - F^K]$	$S^P(t) + t[F^J - F^P]$
t_h	t_h^{ob}	t_h^{ps}	

Figure 11: History effects on oil bank for drainage-imbibition model.

References

[1] Furati, K.M., a. Effects of relative permeability history dependence on two-phase flow in porous media. Preprint.

[2] Furati, K.M., b. The solution of the Riemann problem for a hyperbolic system modeling polymer flooding with hysteresis. *J. Math. Anal. Appl.* To appear.

[3] Lake, L.W., 1989. *Enhanced Oil Recovery.* Prentice Hall, New Jersey.

[4] Pope, G.A., 1980. The application of fractional flow theory to enhanced oil recovery. *Soc. Pet. Engrs. J.*, 20:191–205.