Solution Math 002-01 Quiz 6 (B)

Q.1: If $A = \begin{bmatrix} 2 & 3 & 1 \\ -1 & 2 & 4 \\ 5 & 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 & 1 \\ 3 & -2 & 5 \\ 1 & 2 & 1 \end{bmatrix}$, then find the matrix X such that $2X - 3A = X + 2B$.

Sol: $2X - 3A = X + 2B \quad \Rightarrow X = 3A + 2B = \begin{bmatrix} 6 & 9 & 3 \\ -3 & 6 & 12 \\ 15 & 6 & 9 \end{bmatrix} + \begin{bmatrix} 8 & 6 & 2 \\ 6 & -4 & 10 \\ 2 & 4 & 2 \end{bmatrix} = \begin{bmatrix} 14 & 15 & 5 \\ 3 & 2 & 22 \\ 17 & 10 & 11 \end{bmatrix}$.

Q.2: If $\begin{bmatrix} 2a+1 & 4 \\ 3 & -b+2 \end{bmatrix} + \begin{bmatrix} b+3 & -2 \\ 1 & -3a+2 \end{bmatrix} = \begin{bmatrix} 8 & 2 \\ 4 & 5 \end{bmatrix}$, then find $a + b$.

Sol: $2a + b = 4$ and $-3a - b = 1 \quad \Rightarrow a = -5$ and $b = 14$.

Thus $a + b = -5 + 14 = 9$.

Q.3: If $A = \begin{bmatrix} \frac{m}{3} & 0 & 2 \\ 2 & 1 & 4 \\ 0 & 2 & 1 \end{bmatrix}$, and $B = \begin{bmatrix} -7 & 4 & -2 \\ -2 & 2n & 0 \\ 4 & -2 & 1 \end{bmatrix}$ are inverse of each other, then find $2m + 3n$.

Sol: $-7\left(\frac{m}{3}\right) + 0 + 4(2) = 1 \quad \Rightarrow m = 3$.

$4(2) + 2n(1) - 8 = 1 \quad \Rightarrow n = \frac{1}{2}$. Thus $2m + 3n = 6 + \frac{3}{2} = \frac{15}{2}$.

Q.4: Solve the linear system using matrix inverse method $\begin{array}{r} x - 2y + 3z = 11 \\ 4x + y - z = 4 \\ 2x - y + 3z = 10 \end{array}$.

Sol: The augmented matrix of the coefficient matrix with identity matrix is

$$\left[\begin{array}{ccc|ccc} 1 & -2 & 3 & 1 & 0 & 0 \\ 4 & 1 & -1 & 0 & 1 & 0 \\ 2 & -1 & 3 & 0 & 0 & 1 \end{array}\right] \overset{\substack{-4R_1 + R_2 \\ -2R_1 + R_3}}{\Rightarrow} \left[\begin{array}{ccc|ccc} 1 & -2 & 3 & 1 & 0 & 0 \\ 0 & 9 & -13 & -4 & 1 & 0 \\ 0 & 3 & -3 & -2 & 0 & 1 \end{array}\right]$$

$$\overset{\frac{1}{9}R_2}{\Rightarrow} \left[\begin{array}{ccc|ccc} 1 & -2 & 3 & 1 & 0 & 0 \\ 0 & 1 & \frac{-13}{9} & \frac{-4}{9} & \frac{1}{9} & 0 \\ 0 & 3 & -3 & -2 & 0 & 1 \end{array}\right] \overset{-3R_2 + R_3}{\Rightarrow} \left[\begin{array}{ccc|ccc} 1 & -2 & 3 & 1 & 0 & 0 \\ 0 & 1 & \frac{-13}{9} & \frac{-4}{9} & \frac{1}{9} & 0 \\ 0 & 0 & \frac{4}{3} & \frac{-2}{3} & \frac{-1}{3} & 1 \end{array}\right]$$

$$\overset{\frac{3}{4}R_3}{\Rightarrow} \left[\begin{array}{ccc|ccc} 1 & -2 & 3 & 1 & 0 & 0 \\ 0 & 1 & \frac{-13}{9} & \frac{-4}{9} & \frac{1}{9} & 0 \\ 0 & 0 & 1 & \frac{-1}{2} & \frac{-1}{4} & \frac{3}{4} \end{array}\right] \overset{2R_2 + R_1}{\Rightarrow} \left[\begin{array}{ccc|ccc} 1 & 0 & \frac{1}{9} & \frac{1}{9} & \frac{2}{9} & 0 \\ 0 & 1 & \frac{-13}{9} & \frac{-4}{9} & \frac{1}{9} & 0 \\ 0 & 0 & 1 & \frac{-1}{2} & \frac{-1}{4} & \frac{3}{4} \end{array}\right]$$

$$\overset{\substack{-\frac{1}{9}R_3 + R_1 \\ \frac{13}{9}R_3 + R_2}}{\Rightarrow} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{6} & \frac{1}{4} & -\frac{1}{12} \\ 0 & 1 & 0 & \frac{-7}{6} & \frac{-1}{4} & \frac{13}{12} \\ 0 & 0 & 1 & \frac{-1}{2} & \frac{-1}{4} & \frac{3}{4} \end{array}\right]$$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{1}{6} & \frac{1}{4} & -\frac{1}{12} \\ \frac{-7}{6} & \frac{-1}{4} & \frac{13}{12} \\ \frac{-1}{2} & \frac{-1}{4} & \frac{3}{4} \end{bmatrix} \begin{bmatrix} 11 \\ 4 \\ 10 \end{bmatrix} = \frac{1}{12} \begin{bmatrix} 2 & 3 & -1 \\ -14 & -3 & 13 \\ -6 & -3 & 9 \end{bmatrix} \begin{bmatrix} 11 \\ 4 \\ 10 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$$