Q.1: Verify the identity $\dfrac{1+\sin x}{\cos x}+\dfrac{1}{\tan x+\sec x}=2\sec x$

Sol: $$LHS=\frac{1+\sin x}{\cos x}+\frac{1}{\tan x+\sec x}=\frac{1+\sin x}{\cos x}+\frac{1}{\dfrac{\sin x}{\cos x}+\dfrac{1}{\cos x}}=\frac{1+\sin x}{\cos x}+\frac{\cos x}{\sin x+1}=\frac{(1+\sin x)^2+\cos^2 x}{(1+\sin x)\cos x}$$
$$=\frac{1+\sin^2 x+2\sin x+\cos^2 x}{\cos x\,(1+\sin x)}=\frac{2+2\sin x}{\cos x\,(1+\sin x)}=\frac{2\,(1+\sin x)}{\cos x\,(1+\sin x)}=\frac{2}{\cos x}=2\sec x=RHS$$

Q.2: Find the exact value of $(i)\ \sin(165^o)$, $(ii)\ \tan\left(\frac{19\pi}{12}\right)$

Sol: $$\sin(165^\circ)=\sin(135^\circ+30^\circ)=\sin 135^\circ\cos 30^\circ+\cos 135^\circ\sin 30^\circ$$
$$=\frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2}-\frac{\sqrt{2}}{2}\frac{1}{2}=\frac{\sqrt{2}\left(\sqrt{3}-1\right)}{4}.$$

OR $$\sin(165^\circ)=\sin\left(\frac{330}{2}\right)=\sqrt{\frac{1-\cos 330}{2}}=\sqrt{\frac{1-\dfrac{\sqrt{3}}{2}}{2}}=\frac{\sqrt{2-\sqrt{3}}}{2}$$

$$\tan\left(\tfrac{19\pi}{12}\right)=\tan\left(\frac{15\pi+4\pi}{12}\right)=\tan\left(\frac{15\pi}{12}+\frac{4\pi}{12}\right)$$
$$=\tan\left(\frac{5\pi}{4}+\frac{\pi}{3}\right)=\frac{\tan\left(\dfrac{5\pi}{4}\right)+\tan\left(\dfrac{\pi}{3}\right)}{1-\tan\left(\dfrac{5\pi}{4}\right)\cdot\tan\left(\dfrac{\pi}{3}\right)}=\frac{1+\sqrt{3}}{1-\sqrt{3}}=\frac{\left(1+\sqrt{3}\right)^2}{1-3}$$
$$=\frac{4+2\sqrt{3}}{-2}=-2-\sqrt{3}.$$

Q.3: If $\sin x=\dfrac{3}{5}$, x in Quadrant II and $\sin y=-\dfrac{12}{13}$, y in Quadrant IV. Find $\tan(x+y)$.

Sol: $$\cos x=-\sqrt{1-\sin^2 x}=-\sqrt{1-\frac{9}{25}}=-\frac{4}{5}$$
$$\cos y=\sqrt{1-\sin^2 y}=\sqrt{1-\frac{144}{169}}=\frac{5}{13}.$$
$$\tan(x+y)=\frac{\tan x+\tan y}{1-\tan x\tan y}=\frac{-\dfrac{3}{4}-\dfrac{12}{5}}{1-\dfrac{3}{4}\dfrac{12}{5}}=\frac{\dfrac{-15-48}{20}}{\dfrac{20-36}{20}}=\frac{63}{16}$$

Q.4: Evaluate $\sin\left(\dfrac{5\pi}{12}\right)$ and $\cos(67.5^o)$

Sol: $$\sin\left(\frac{5\pi}{12}\right)=\sin\left(\frac{3\pi}{12}+\frac{2\pi}{12}\right)=\sin\left(\frac{\pi}{4}+\frac{\pi}{6}\right)=\sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right)+\cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)$$
$$=\frac{1}{4}\sqrt{2}+\frac{1}{4}\sqrt{2}\sqrt{3}=\frac{\sqrt{2}\left(1+\sqrt{3}\right)}{4}.$$

OR $$\sin\left(\frac{5\pi}{12}\right)=\sin\left(\frac{\dfrac{5\pi}{6}}{2}\right)=\sqrt{\frac{1-\cos\left(\dfrac{5\pi}{6}\right)}{2}}=\sqrt{\frac{1+\dfrac{\sqrt{3}}{2}}{2}}=\frac{\sqrt{2+\sqrt{3}}}{2}$$

$$\cos(67.5^\circ)=\cos\left(\frac{135^\circ}{2}\right)=\sqrt{\frac{1+\cos(135^\circ)}{2}}=\sqrt{\frac{1-\dfrac{\sqrt{2}}{2}}{2}}=\frac{\sqrt{2-\sqrt{2}}}{2}.$$

Q.5: Verify the identity $\sin(3x)=3\sin x-4\sin^3 x$

Sol: $$\sin(3x)=\sin(x+2x)=\sin x\cos(2x)+\cos x\sin(2x)$$
$$=\sin x\left(1-2\sin^2 x\right)+\cos x\,(2\sin x\cos x)$$
$$=\sin x-2\sin^3 x+2\sin x\cos^2 x$$
$$=\sin x-2\sin^3 x+2\sin x\left(1-\sin^2 x\right)$$

$$= \sin x - 2\sin^3 x + 2\sin x - 2\sin^3 2$$
$$= 3\sin x - 4\sin^3 x.$$