

Name: Solution

Serial # _____

1. Write the vector $h = (2, -7, 9)$ as a linear combination of the vectors u , v , and w where $u = (1, -1, 2)$, $v = (3, 0, 1)$, $w = (1, -2, 2)$.

$$h = c_1 u + c_2 v + c_3 w$$

$$\begin{aligned} (2, -7, 9) &= c_1(1, -1, 2) + c_2(3, 0, 1) + c_3(1, -2, 2) \\ &= (c_1 + 3c_2 + c_3, -c_1 - 2c_3, 2c_1 + c_2 + 2c_3) \end{aligned}$$

$$\Rightarrow \begin{cases} c_1 + 3c_2 + c_3 = 2 \\ -c_1 - 2c_3 = -7 \\ 2c_1 + c_2 + 2c_3 = 9 \end{cases}$$

Solving this system, we get $c_1 = 3$, $c_2 = -2$, $c_3 = 2$

$$\therefore (2, -7, 9) = 3(1, -1, 2) - 2(3, 0, 1) + 2(1, -2, 2)$$

$$h = 3u - 2v + 2w$$

2. Choose the correct answer:

The set $S = \{(3, 1, 0, 2), (1, 5, 11, 0), (3, 3, 3, 1)\}$ in \mathbb{R}^4 is:

- (a) a basis for \mathbb{R}^4 (b) a spanning set for \mathbb{R}^4 (c) not a basis (d) linearly dependent

$|S| = 3 < 4$ i.e. 3 vectors in \mathbb{R}^4 will not span $\mathbb{R}^4 \Rightarrow S$ is not a basis.

3. Show that the set of functions $\{\cos x, \sin x\}$ is linearly independent.

$$\begin{aligned} W(\cos x, \sin x) &= \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = \cos^2 x + \sin^2 x \\ &= 1 \\ &\neq 0 \end{aligned}$$

$\therefore \{\cos x, \sin x\}$ is linearly indep.