Name: Solution

Math 241 – Quiz # 3a

Sr #: _____

1. Determine whether the vectors u, v and w are linearly independent or not, where $u = (1, 0, 3)$, $v = (2, 1, 0)$, $w = (3, 2, -1)$.

$$c_1 u + c_2 v + c_3 w = 0$$

$$c_1(1,0,3) + c_2(2,1,0) + c_3(3,2,-1) = (0,0,0)$$

$$\Rightarrow \begin{cases} c_1 + 2c_2 + 3c_3 = 0 \\ c_2 + 2c_3 = 0 \\ 3c_1 - c_3 = 0 \end{cases} \qquad \Rightarrow A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 3 & 0 & -1 \end{bmatrix}$$

$$\det(A) = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 3 & 0 & -1 \end{vmatrix} = 2 \neq 0$$

$\Rightarrow$ The system has only the trivial solution.

i.e. $c_1 = c_2 = c_3 = 0$

$\Rightarrow$ v_1, u, w are linearly independent.

2. Choose the correct answer:

The set $S = \{(2,0,1,3),(1,3,3,3),(0,11,5,1)\}$ in R^4 is:

(a) a basis for R^4 (b) a spanning set for R^4 (c) not a basis (d) linearly dependent

Since $|S| = 3$ in $\mathbb{R}^4$, $3 < 4 \Rightarrow S$ does not span $\mathbb{R}^4$

$\Rightarrow S$ is not a basis.

3. Show that the set of functions $\{\sin x, \cos x\}$ is linearly independent.

$$W(\sin x, \cos x) = \begin{vmatrix} \sin x & \cos x \\ \cos x & -\sin x \end{vmatrix} = -\sin^2 x - \cos^2 x$$

$$= -(\sin^2 x + \cos^2 x) = -1 \neq 0$$

$\therefore \{\sin x, \cos x\}$ is linearly indep.